King Saud University | College of Sciences | Department of Mathematics

Second Examination | Math 132 | Semester 2 (1443) | Time:1H30

Exercise 1: (6 marks)

1. Consider the set $A := \{1,2,\{1\},\{2\},\{1,2,\emptyset\},\{1,\{1\}\},\{2,\{2\}\},\emptyset,\{\emptyset\}\}$.
   Determine whether each of the following five statements is true or false.
   (Justify your answer). (5 marks)
   S_1: "$\{1,2\} \in A$", S_2: "$\{1,2,\emptyset\} \subseteq A$", S_3: "$\{1,\{1\}\} \subseteq A$"
   S_4: "$\{1,\{\emptyset\}\} \subseteq A$", S_5: "$A \cap \{1,2,\emptyset,\{\{1\},\{2\}\}\} = \{1,2\}$".

2. Let X and Y be two sets such that $X - Y = \{0,2,4,6,8\}$, $Y - X = \{1,3,5,7,9\}$ and $X \cap Y = \{10,11,20\}$. Find X and Y. (1 mark)

Exercise 2: (19 marks)

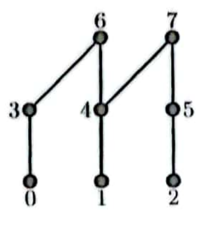
1. Let R be the relation from $A = \{3,4,5,6,7\}$ to $B = \{1,2,3,4,5\}$ defined by
$$aRb \Longleftrightarrow a - b = 3$$
   (a) List all ordered pair of R. (1 mark)
   (b) Find the domain and the image of R. (1 mark)
   (c) Represent R by a matrix. (1 mark)

2. Let $S = \{(a,a),(a,c),(b,b),(b,d),(c,a),(c,c)\}$ be a relation on $C = \{a,b,c,d\}$.
   (a) Represent S by a digraph. (1 mark)
   (b) Find S^2. (2 marks)
   (c) Find $\overline{S \cup S^{-1}}$. (2 marks)

3. Let T be the relation on $\mathbb{Z} \setminus \{0\}$ defined by:
$$mTn \Longleftrightarrow mn > 0$$
   Determine whether T is reflexive, symmetric, antisymmetric or transitive. (4 marks).

4. Let $E = \{(1,1),(1,3),(1,5),(2,2),(2,4),(3,1),(3,3),(3,5),(4,2),(4,4),(5,1),(5,3),(5,5)\}$ be a relation on $\{1,2,3,4,5\}$.
   (a) Show that E is an equivalence relation on $\{1,2,3,4,5\}$. (3 marks)
   (b) Find all distinct equivalence classes of E. (1 mark)

5. Let P be the partial ordering relation defined on the set $\{0,1,2,3,4,5,6,7\}$ represented by the following Hasse diagram.



   (a) List all ordered pair of P. (2 marks)
   (b) Determine whether P is a totally ordering relation on $\{0,1,2,3,4,5,6,7\}$. (1 mark)

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Exercice 1:

1)

S_1: false because $\{1,2\}$ not an element of A. 1

S_2: $\{1,2,\phi\} \subseteq A$: True , $1 \in A$; $2 \in A$; $\phi \in A$. 1

S_3: $\{1,\{1\}\} \subseteq A$ True : $1 \in A$, $\{1\} \in A$. 1

S_4: True : $1 \in A$; $\{\phi\} \in A$. 1

S_5: $A \cap \{1,2,\phi,\{\{1\}\},\{2\}\} = \{1,2,\phi\}$, false. 1

2).

$X = \{0,2,4,6,8,10,11,20\}$.

$Y = \{1,3,5,7,9,10,11,20\}$. 1

Exercice 2:

1) a). $R = \{(4,1),(5,2),(6,3),(7,4)\}$. 1

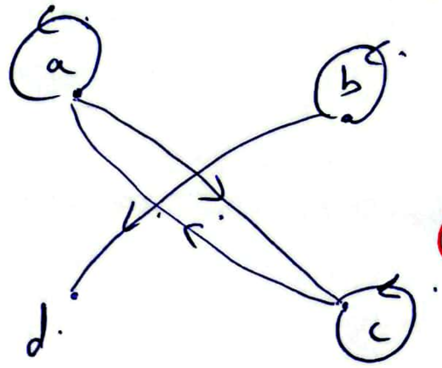
b) domain$(R) = \{4,5,6,7\}$.

image$(R) = \{1,2,3,4\}$. 1

c).

$$M_R = \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$ 1

2).



1.

b) $S^2 = S \circ S$

$$M_S = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$M_{S^2} = M_S \otimes M_S = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = M_S$$

2

$\Rightarrow S^2 = S = \{(a,a),(a,c),(b,b),(b,d),(c,a),(c,c)\}.$

c).

$S^{-1} = \{(a,a),(c,a),(b,b),(d,b),(a,c),(c,c)\}.$

$S \cup S^{-1} = \{(a,a),(a,c),(b,b),(b,d),(c,a),(c,c),(d,b)\}.$ 2

$\overline{S \cup S^{-1}} = \{(a,b),(a,d),(b,a),(b,c),(c,b),(c,d),(d,a),(d,c),(d,d)\}.$

3)

- Let $a \in \mathbb{Z} \setminus \{0\}$; $a.a = a^2 > 0 \Rightarrow aTa \Rightarrow T$ is reflexive.
- Let $a,b \in \mathbb{Z} \setminus \{0\}$; $aRb \Rightarrow a.b > 0 \Rightarrow b.a > 0 \Rightarrow bRa$
  $\Rightarrow R$ is symmetric.
  $\Rightarrow R$ is not anti_symmetric.
- ~~Let $a,b,c \in \mathbb{Z} \setminus \{0\}$; aRb and $bRc \Rightarrow a.b>0$ and $b.c>0$.~~

~~$a.b>0 \Rightarrow a>0$ and $b>0$ or $a<0$ and $b<0$.~~

~~$\cdot\ a>0$ and $b>0$;~~

~~$b.c>0 \Rightarrow c>0 \Rightarrow$.~~ ok. - 4

~~but~~

$\dfrac{a>0 \text{ and } b>0}{b.c>0 \Rightarrow c>0 \Rightarrow ac>0 \Rightarrow aRc.}$

$\dfrac{a<0 \text{ and } b<0}{b.c>0 \Rightarrow c<0 \Rightarrow a.c>0 \Rightarrow aRc}$ $\Bigg\} \Rightarrow R$ is transitive.

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4)

a) • $(1,1),(2,2),(3,3),(4,4),(5,5) \in E \Rightarrow E$ is reflexive. ①

• $(1,3)\in E, (3,1)\in E$.
$(1,5)\in E, (5,1)\in E$.
$(2,4)\in E; (4,2)\in E$.
$(3,5)\in E; (5,3)\in E$.
$\Rightarrow E$ is symmetric. ②

• $(1,3)\in E; (3,5)\in E \Rightarrow (1,5)\in E$;
$(3,1)\in E \Rightarrow (1,1)\in E$.
$(1,5)\in E; (5,3)\in E \Rightarrow (1,3)\in E$
$(5,1)\in E \Rightarrow (1,1)\in E$.
$(3,1)\in E; (1,3)\in E \Rightarrow (3,3)\in E$.
$(1,5)\in E \Rightarrow (3,5)\in E$.
$(3,5)\in E; (5,1)\in E \Rightarrow (3,1)\in E$.
$(5,3)\in E \Rightarrow (3,3)\in E$
$\Rightarrow E$ is transitive. ③

①, ② and ③ $\Rightarrow E$ is an equivalence relation on $\{1,2,3,4,5\}$.

3

b) The classes of E are : $\{1,3,5\}, \{2,4\}$. 1

5).

a) $P=\{(0,0),(0,3),(0,6),(1,1),(1,4),(1,6),(1,7),(2,2),(2,5),(2,7),$
$(3,3),(3,6),(4,4),(4,6),(4,7),(5,5),(5,7),(6,6),(7,7)\}$. 2

b) No.
$3 \not{P} 4 ; 4 \not{P} 3$ 1