physics for scientists and engineers (serway)

Dr.WAFAA MUJAMAMMI

Chapter 23
Electric Fields

CHAPTER OUTLINE

23.3 Coulomb's Law

23.4 The Electric Field

23.6 Electric Field Lines

23.7 Motion of Charged Particles in a Uniform Electric Field

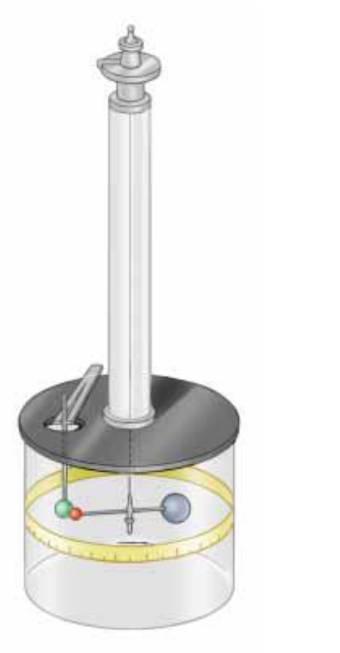
electric charge has the following important properties:

- There are two kinds of charges in nature; charges of opposite sign attract one Properties of electric charge
- another and charges of the same sign repel one another.
- Total charge in an isolated system is conserved.
- Charge is quantized Q=nq.

23.3 Coulomb’s Law

Charles Coulomb (1736–1806) measured the magnitudes of the electric forces between charged objects using the torsion balance

$$F_e \propto 1/r^2.$$



Torsion balance

23.3 Coulomb’s Law

From Coulomb’s experiments, we can generalize the following properties of the electric force between two stationary charged particles.

The electric force

- is inversely proportional to the square of the separation r between the particles and directed along the line joining them;
- is proportional to the product of the charges $q1$ and $q2$ on the two particles;
- is attractive if the charges are of opposite sign and repulsive if the charges have the same sign;
- is a conservative force.

23.3 Coulomb's Law

From experimental observations on the electric force, we can express Coulomb's law as an equation giving the magnitude of the electric force (sometimes called the *Coulomb force*) between two point charges:

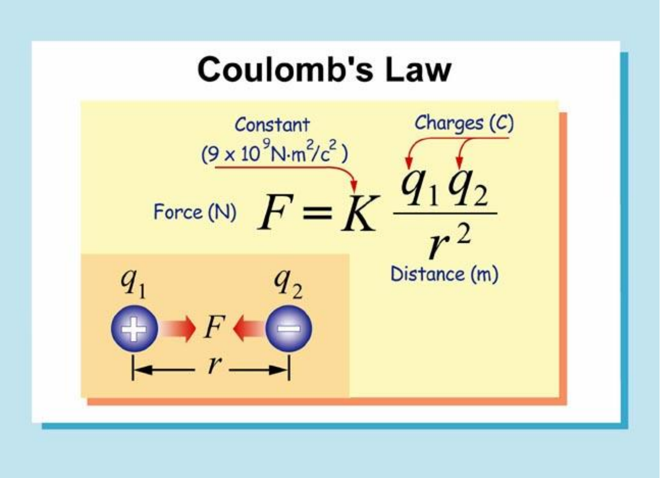
$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

Where:

r: is separation between the particles and directed along the line joining them

23.3 Coulomb's Law

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$



q: is a charge

ke is a constant called the Coulomb constant

$$k_e = 8.9875 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

23.3 Coulomb's Law

This constant is also written in the form

$$k_e = \frac{1}{4\pi\epsilon_0}$$

where the constant ϵ_0 (lowercase Greek epsilon) is known as the **permittivity of free space** and has the value

$$\epsilon_0 = 8.854\,2 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

The smallest unit of charge *e*

is the charge on an electron (-*e*)
or a proton (+*e*) and has a magnitude

$$e = 1.602\,19 \times 10^{-19}\ \mathrm{C}$$

23.3 Coulomb's Law

Example 23.1 **The Hydrogen Atom**

The electron and proton of a hydrogen atom are separated(on the average) by a distance of approximately 5.3 x 10^{-11} m. Find the magnitudes of the electric force

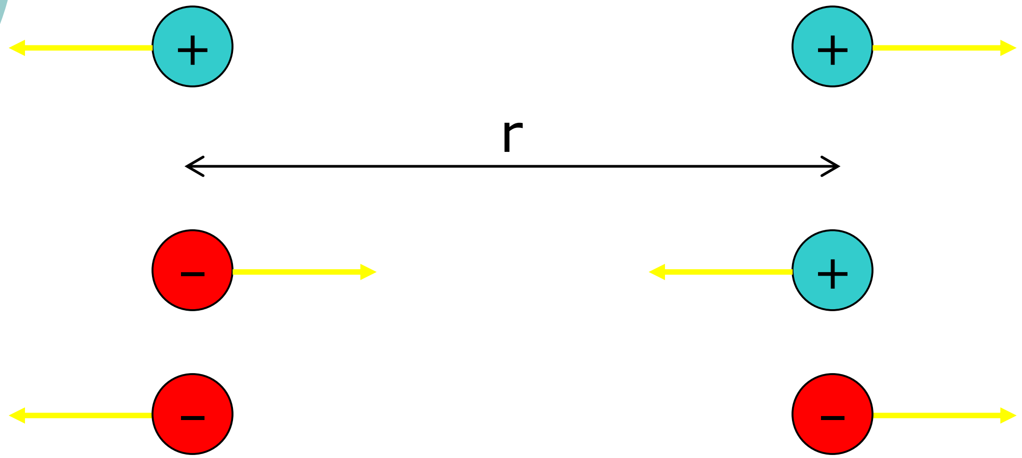
Solution From Coulomb's law, we find that the magnitude of the electric force is

$$F_e = k_e \frac{|e||-e|}{r^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\,\frac{(1.60 \times 10^{-19}\text{ C})^2}{(5.3 \times 10^{-11}\text{ m})^2}$$

$$= \boxed{8.2 \times 10^{-8}\text{ N}}$$

Coulomb Law Illustrated

- Like charges repel
- Unlike charges attract



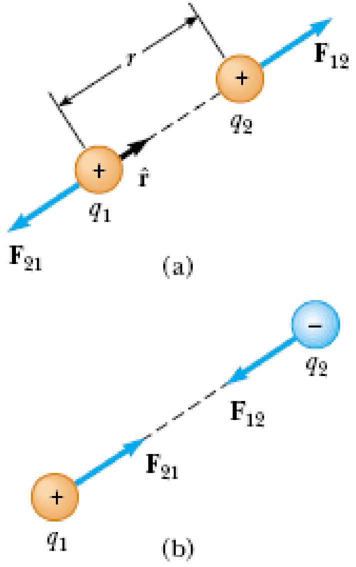
If charges are of same magnitude (and same separation), all the forces will be the same magnitude, with different directions.

23.3 Coulomb's Law

When dealing with Coulomb's law, you must remember that force is a vector quantity and must be treated accordingly. The law expressed in vector form for the electric force exerted by a charge q_1 on a second charge q_2, written $\mathbf{F}_{12}$, is

Vector form of Coulomb's law

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$



where $\hat{\mathbf{r}}$ is a unit vector directed from q_1 toward q_2

Coulomb Force Law, Qualitatively

- Double one of the charges
  - force doubles
- Change sign of one of the charges
  - force changes direction
- Change sign of *both* charges
  - force stays the same
- Double the distance between charges
  - force four times weaker
- Double both charges
  - force four times stronger

23.3 Coulomb's Law

When more than two charges are present, the force between any pair of them is given by Equation

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

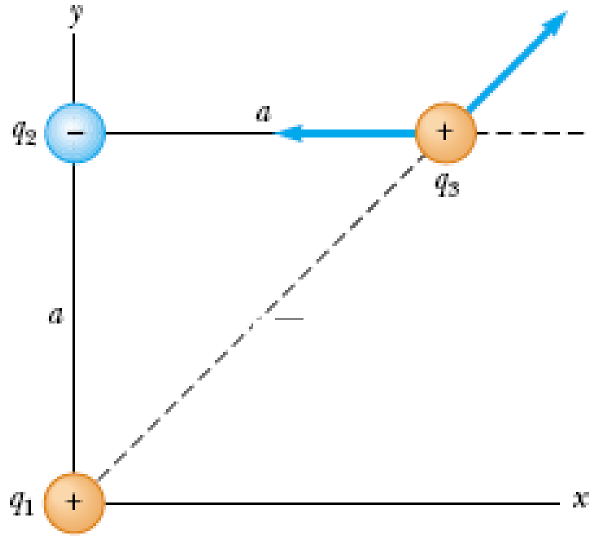
Therefore, the resultant force on any one of them equals the vector sum of the forces exerted by the various individual charges. For example, if four charges are present, then the resultant force exerted by particles 2, 3, and 4 on particle 1 is

$$\mathbf{F}_1 = \mathbf{F}_{21} + \mathbf{F}_{31} + \mathbf{F}_{41}$$

23.3 Coulomb's Law

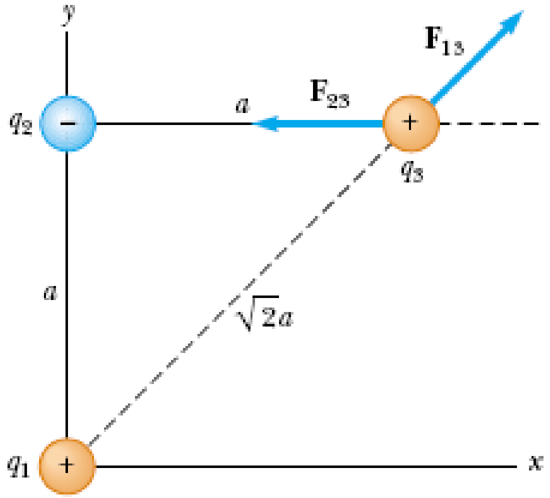
Example 23.2 **Find the Resultant Force**

Consider three point charges located at the corners of a right triangle as shown in Figure where $q_1 = q_3 = 5.0\ \mu C$, $q_2 = -2.0\ \mu C$, and $a = 0.10$ m. Find the resultant force exerted on q_3.



23.3 Coulomb's Law

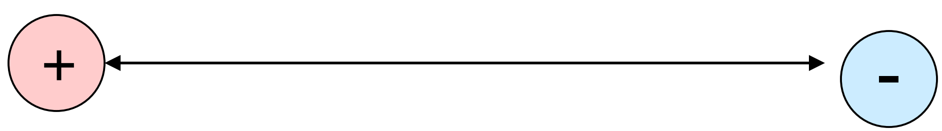
Solution : see the book



$$\mathbf{F}_3 = \boxed{(-1.1\hat{\mathbf{i}} + 7.9\hat{\mathbf{j}})\ \mathrm{N}}$$

Coulomb’s Law: Ex 1

Determine the magnitude of the force between a proton and an electron in a hydrogen atom. Assume the distance from the electron to the nucleus is 0.53×10^{-10} m.



Coulomb's Law: Ex 1

$$F = k \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{(9.0 \times 10^{9}\ N\text{-}m^2/C^2)(1.602 \times 10^{-19}\ C)(1.602 \times 10^{-19}\ C)}{(0.53 \times 10^{-10}\ m)^2}$$

$$\mathbf{F = 8.2 \times 10^{-8}\ N}$$

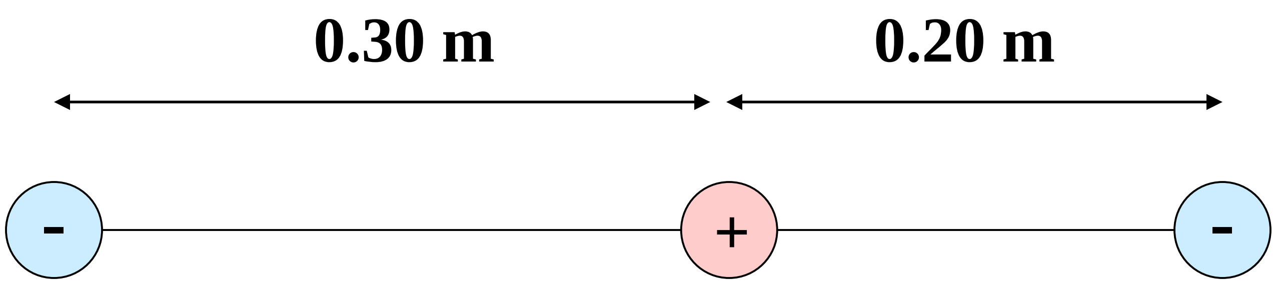
Coulomb's Law: Ex 2

What is the force between an electron and the three protons in a Li atom if the distance is about 1.3×10^{-10}?

(ANS: 3.9×10^{-8} N)

Coulomb's Law: Ex 3

Three charged particles are arranged in a straight line as shown in the diagram. Calculate the net force on particle 3.



Q_1 = -8.0 μC Q_2 = +3.0 μC Q_3 = -4.0 μC

Coulomb's Law: Ex 3

$$F = k\,\frac{Q_1 Q_2}{r^2}$$

$$F_{13} = \frac{(9.0 \times 10^{9}\ \text{N-m}^2/\text{C}^2)(4.0\times 10^{-6}\ \text{C})(8.0\times10^{-6}\ \text{C})}{(0.50\ \text{m})^2}$$

F_{13} = 1.2 N (Repulsive to the right)

$$F_{23} = \frac{(9.0 \times 10^{9}\ \text{N-m}^2/\text{C}^2)(4.0\times 10^{-6}\ \text{C})(3.0\times10^{-6}\ \text{C})}{(0.20\ \text{m})^2}$$

F_{23} = 2.7 N (Attractive to the left)

F_{net} = $\mathbf{F_{13}}$ **-** $\mathbf{F_{23}}$

F_{net} = 1.2 N – 2.7 N = **-1.5 to the left**

Coulomb's Law: Ex 4

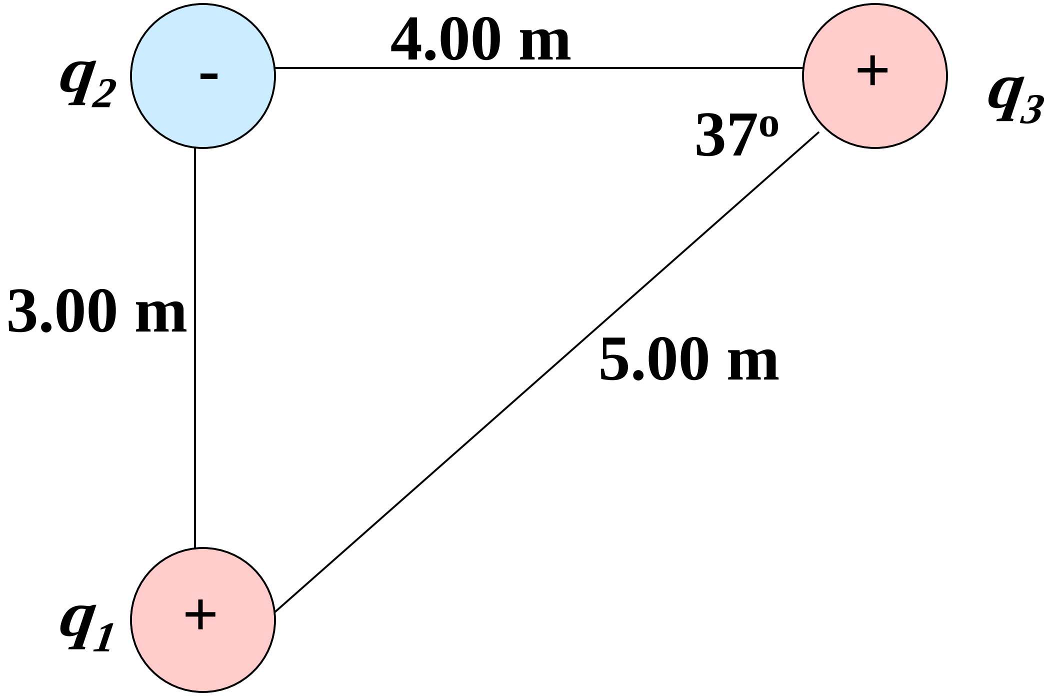
What is the resultant force on charge q_3 if the charges are arranged as shown below. The magnitudes of the charges are:

$q_1 = +6.00 \times 10^{-9}$ C

$q_2 = -2.00 \times 10^{-9}$ C

$q_3 = +5.00 \times 10^{-9}$ C

Coulomb's Law: Ex 4



Coulomb's Law: Ex 4

First calculate the forces on q_3 separately:

$$F_{13} = k \frac{Q_1 Q_3}{r^2}$$

$$F_{13} = \frac{(9.0 \text{ X } 10^9 \text{ N-m}^2/\text{C}^2)(6.00 \text{ X } 10^{-9} \text{ C})(5.00 \text{ X } 10^{-9} \text{ C})}{(5.00\text{m})^2}$$

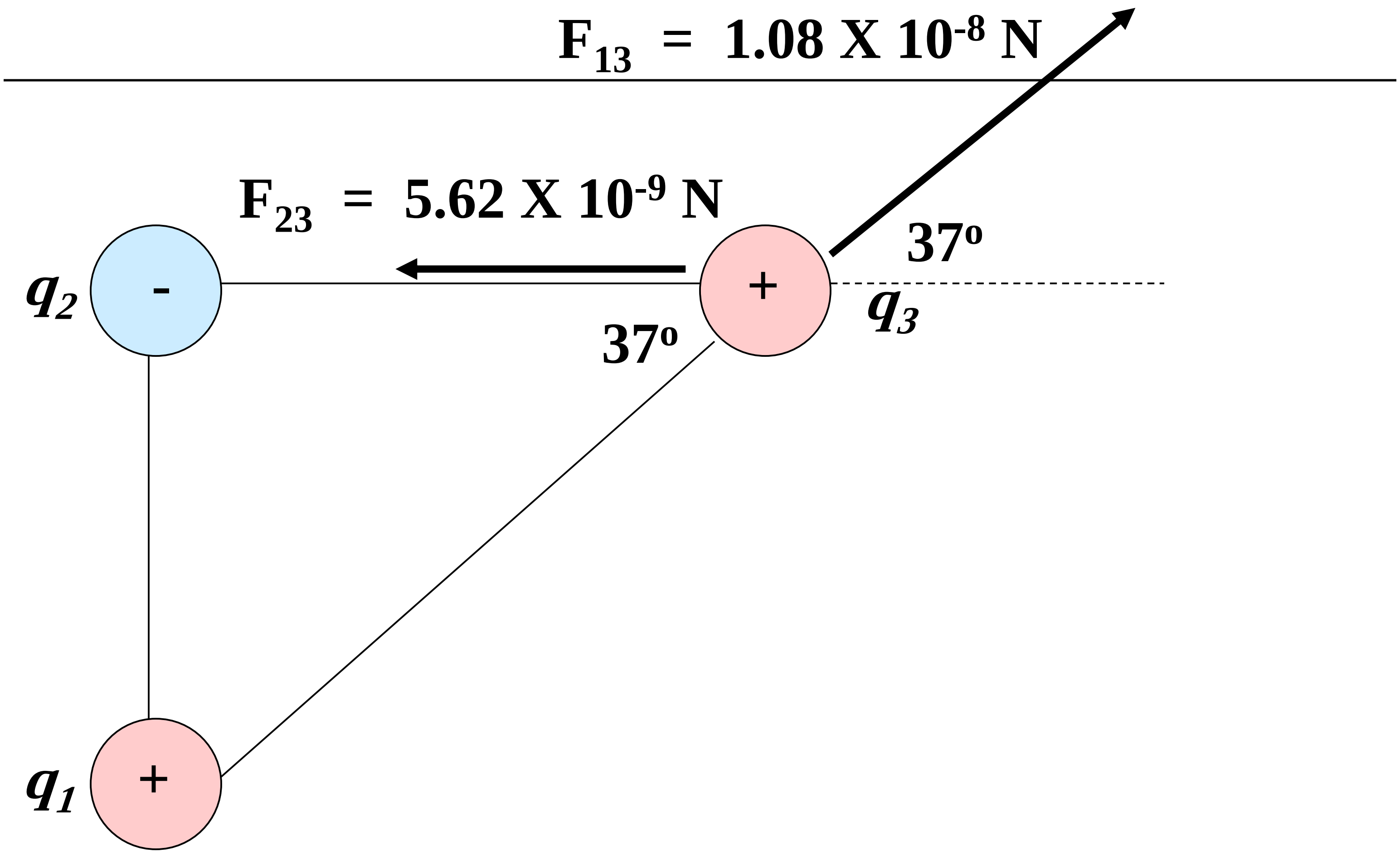
$$\mathbf{F_{13} = 1.08 \text{ X } 10^{-8} \text{ N}}$$

Coulomb's Law: Ex 4

$$F_{23} = k \frac{Q_2 Q_3}{r^2}$$

$$F_{23} = \frac{(9.0 \text{ X } 10^9 \text{ N-m}^2/\text{C}^2)(2.00 \text{ X } 10^{-9} \text{ C})(5.00 \text{ X } 10^{-9} \text{ C})}{(4.00\text{m})^2}$$

$$\mathbf{F_{23} = 5.62 \text{ X } 10^{-9} \text{ N}}$$



F13 = 1.08 X 10-8 N
F23 = 5.62 X 10-9 N
37o
37o
q2
-
+
q3
+
q1

Coulomb's Law: Ex 4

$F_{13x} = \ F_{13}\cos 37^{o} = (1.08 \text{ X } 10^{-8} \text{ N})\cos 37^{o}$

$\mathbf{F_{13x} = 8.63 \text{ X } 10^{-9} \text{ N}}$

$F_{13y} = \ F_{13}\sin 37^{o} = (1.08 \text{ X } 10^{-8} \text{ N})\sin 37^{o}$

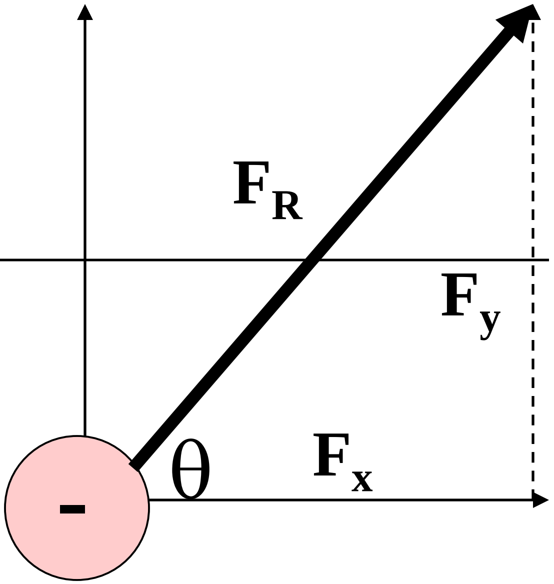
$\mathbf{F_{13y} = 6.50 \text{ X } 10^{-9} \text{ N}}$

$F_x = F_{23} + F_{13x}$

$\mathbf{F_x}$ $= -5.62 \text{ X } 10^{-9} \text{ N} + 8.63 \text{ X } 10^{-9} \text{ N} =$ $\mathbf{3.01 \text{ X } 10^{-9} \text{ N}}$

$\mathbf{F_y}$ $= F_{13y} =$ $\mathbf{6.50 \text{ X } 10^{-9} \text{ N}}$

Coulomb's Law: Ex 4



$F_x = 3.01 \times 10^{-9}$ N

$F_y = 6.50 \times 10^{-9}$ N

$F_R = \sqrt{(3.01 \times 10^{-9}\ \text{N})^2 + (6.50 \times 10^{-9}\ \text{N})^2}$

$\mathbf{F_R = 7.16 \times 10^{-9}\ N}$

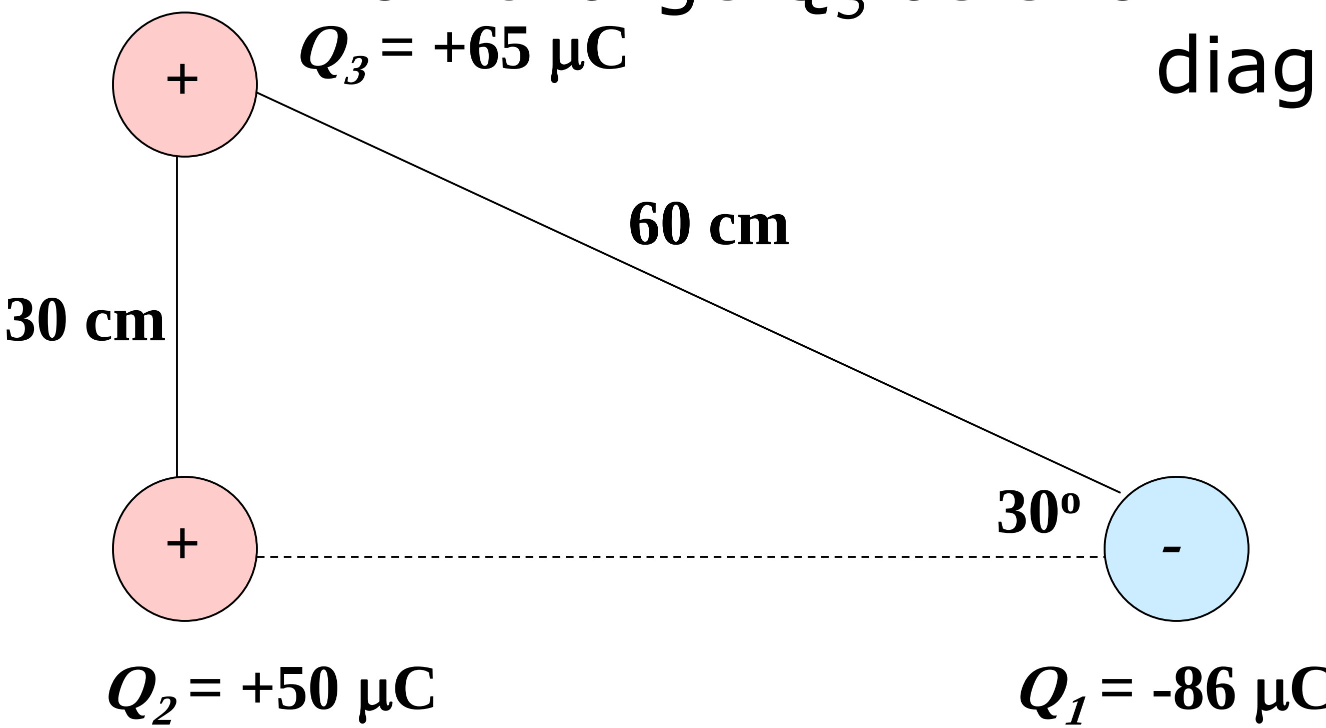
$\sin\theta = F_y/F_R$

$\sin\theta = (6.50 \times 10^{-9}\ \text{N})/(7.16 \times 10^{-9}\ \text{N})$

$\boldsymbol{\theta = 64.7^{\circ}}$

Coulomb's Law: Ex 5

Calculate the net electrostatic force on charge Q_3 as shown in the diagram:



Coulomb's Law: Ex 5

First calculate the forces on Q_3 separately:

$$F_{13} = k \frac{Q_1 Q_3}{r^2}$$

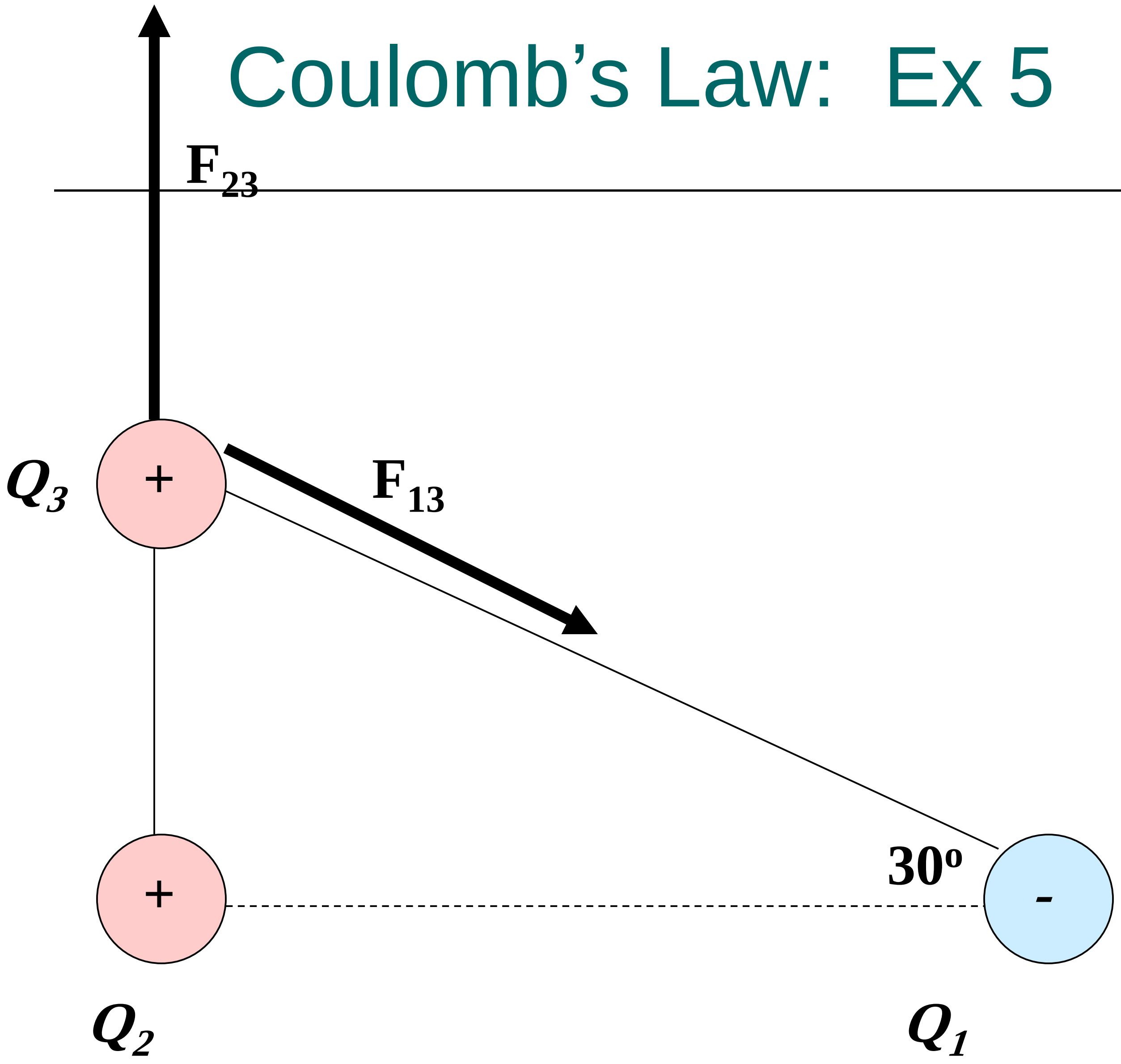
$$F_{13} = \frac{(9.0 \text{ X } 10^9 \text{ N-m}^2/\text{C}^2)(65 \text{ X } 10^{-6} \text{ C})(86 \text{ X } 10^{-6} \text{ C})}{(0.60 \text{ m})^2}$$

$\mathbf{F_{13} = 140\ N}$

$$F_{23} = \frac{(9.0 \text{ X } 10^9 \text{ N-m}^2/\text{C}^2)(65 \text{ X } 10^{-6} \text{ C})(50 \text{ X } 10^{-6} \text{ C})}{(0.30 \text{ m})^2}$$

$\mathbf{F_{23} = 330\ N}$

Coulomb's Law: Ex 5



Coulomb's Law: Ex 5

$F_{13x} = \quad F_{13}\cos 30^o = (140\ N)\cos 30^o$

$\mathbf{F_{13x} = 120\ N}$

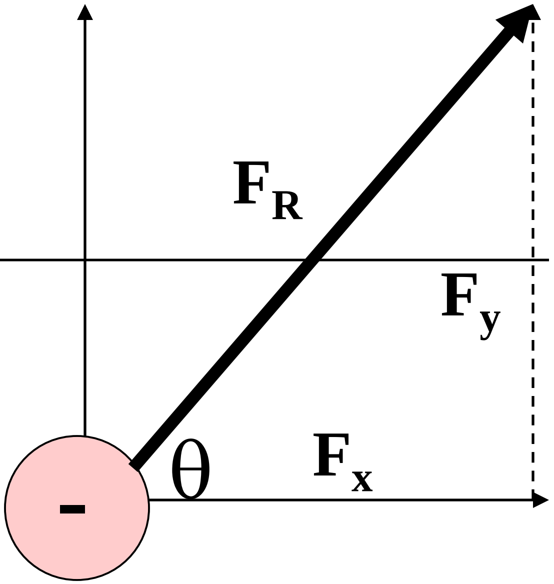
$F_{13y} = \quad -F_{13}\sin 30^o = (140\ N)\sin 30^o$

$\mathbf{F_{13y} = -70\ N}$

$\mathbf{F_x =}\ F_{13x} = \mathbf{120\ N}$

$\mathbf{F_7} = 330\ N - 70\ N \ = \mathbf{260\ N}$

Coulomb's Law: Ex 5



$F_x = 120\ N$

$F_y = 260\ N$

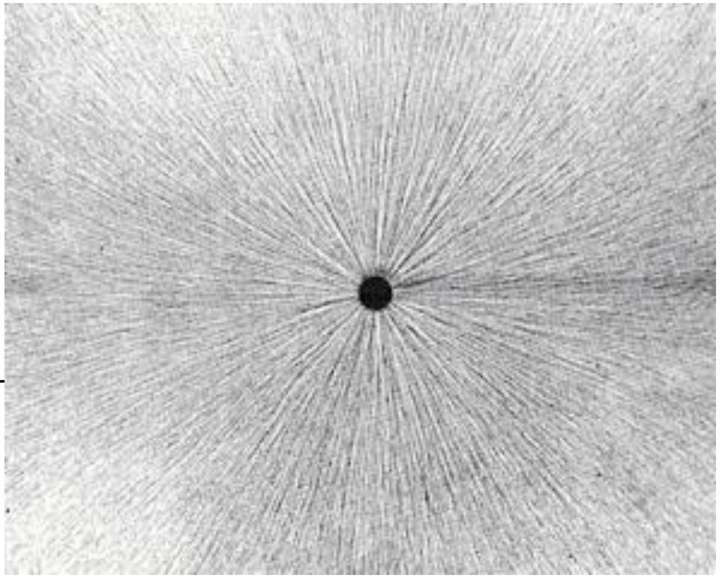
$F_R = \sqrt{(120\ N)^2 + (330\ N)^2}$

$\mathbf{F_R = 290\ N}$

$\sin\theta = F_y/F_R$

$\sin\theta = (260\ N)/\ (290\ N)$

$\boldsymbol{\theta = 64^o}$

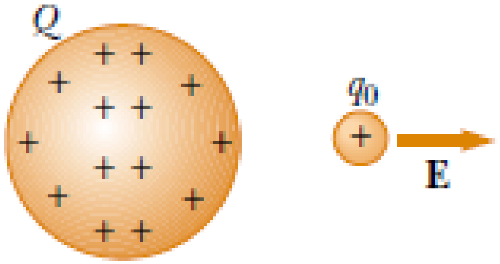


23.4 The Electric Field

23.4 The Electric Field

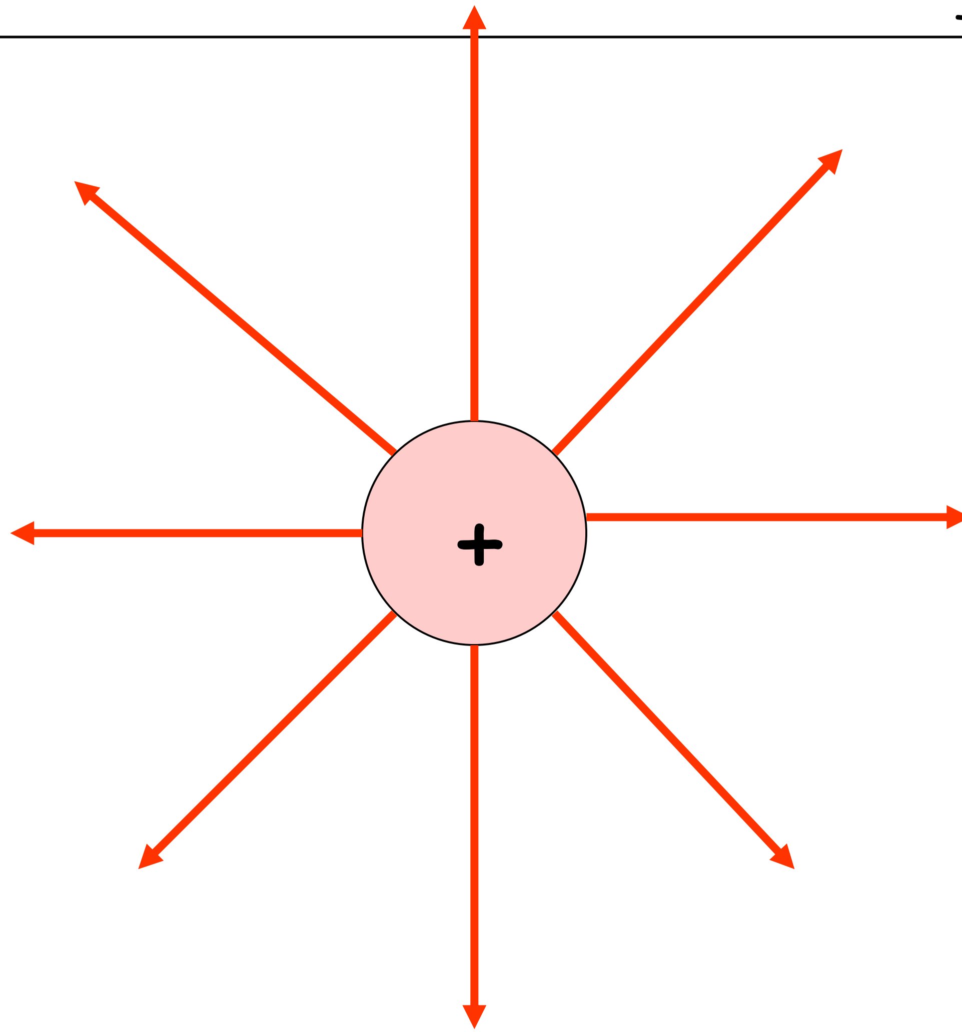
the electric field vector E at a point in space is defined as the electric force $\mathbf{F}_e$ acting on a positive test charge q_0 placed at that point divided by the test charge:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$

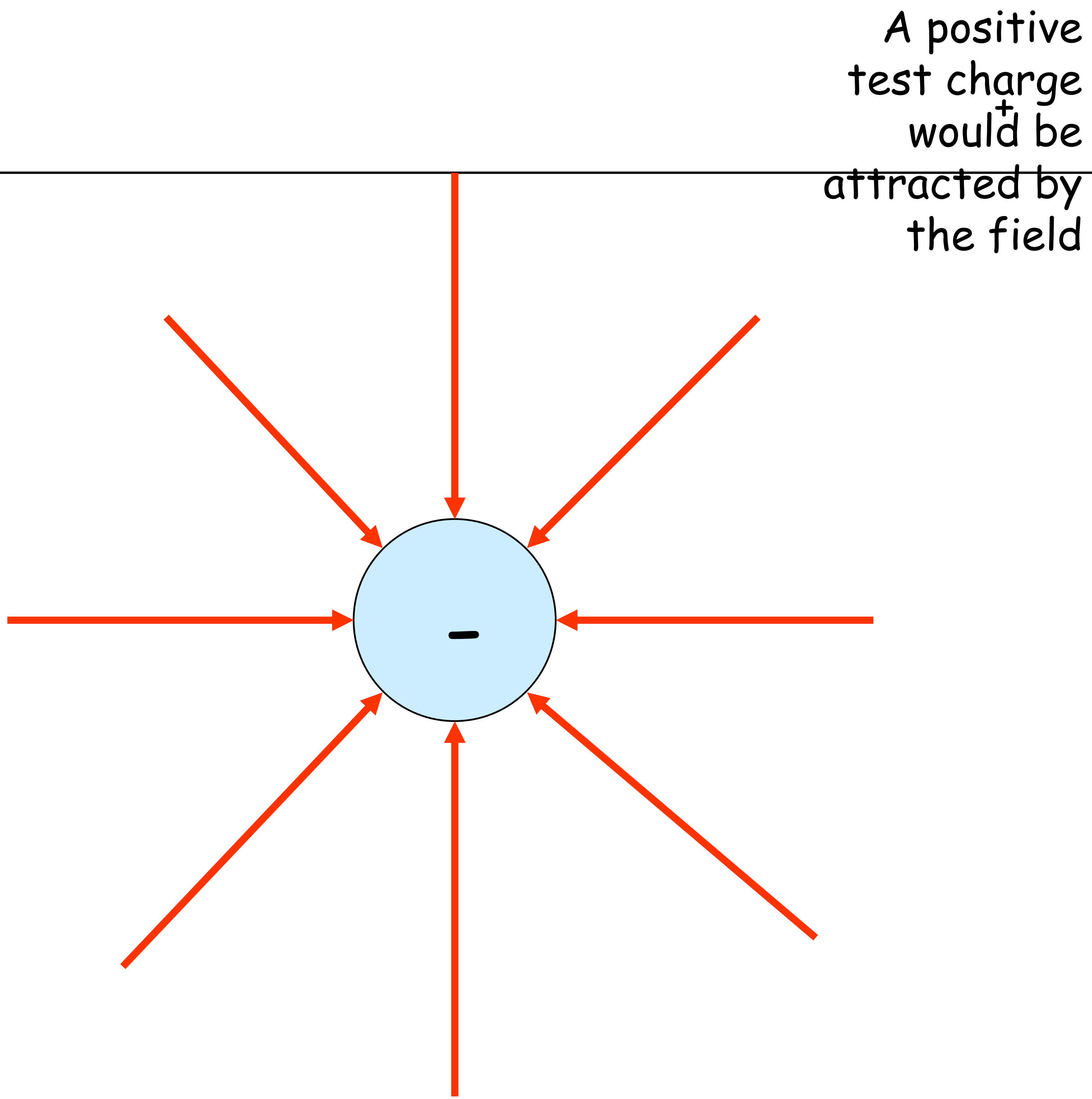


Proton

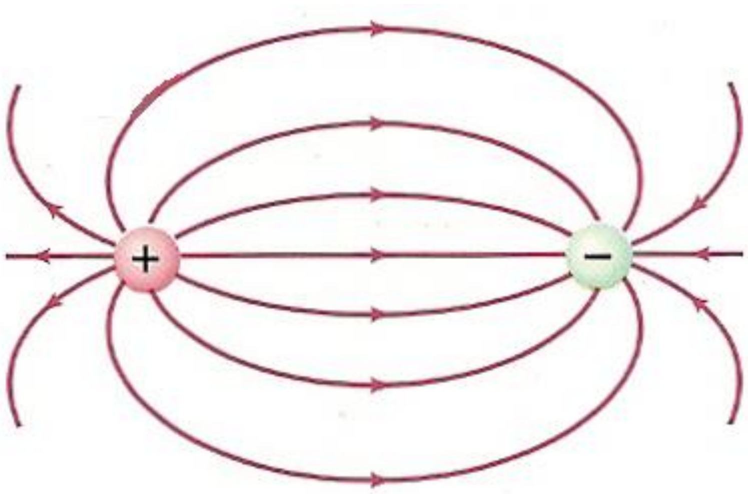
A positive test charge would be repelled by the field

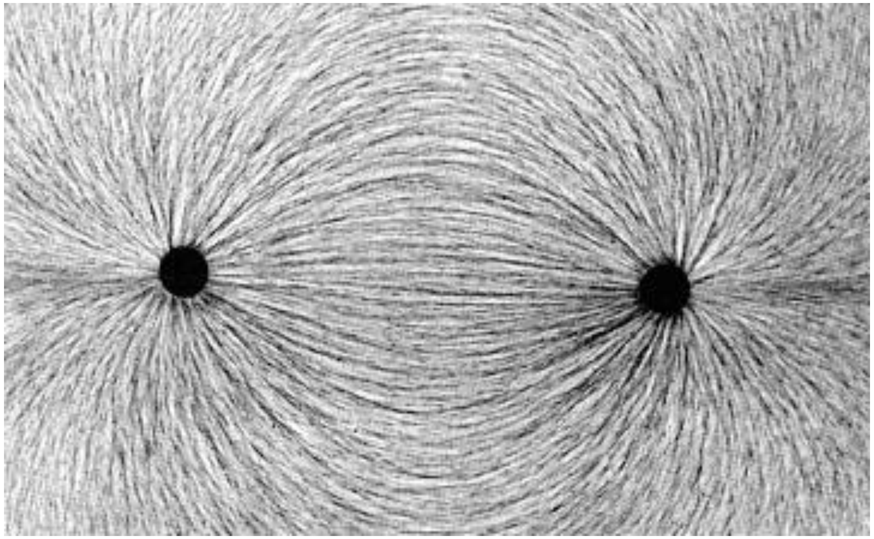


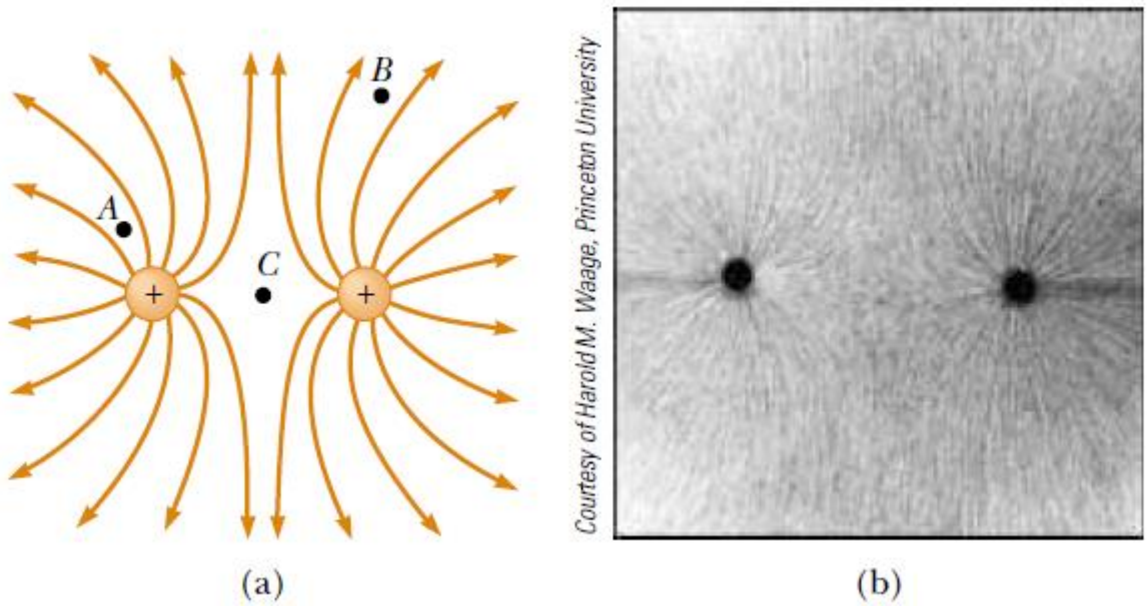
Electron



Opposite charges attract

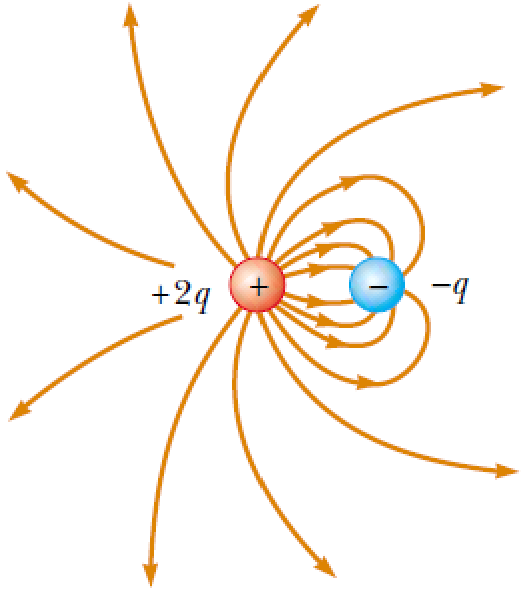






(a) The electric field lines for two positive point charges. (The locations *A, B, and C are discussed in Quick Quiz 23.7.) (b) Pieces of thread suspended in oil,* which align with the electric field created by two equal-magnitude positive charges.

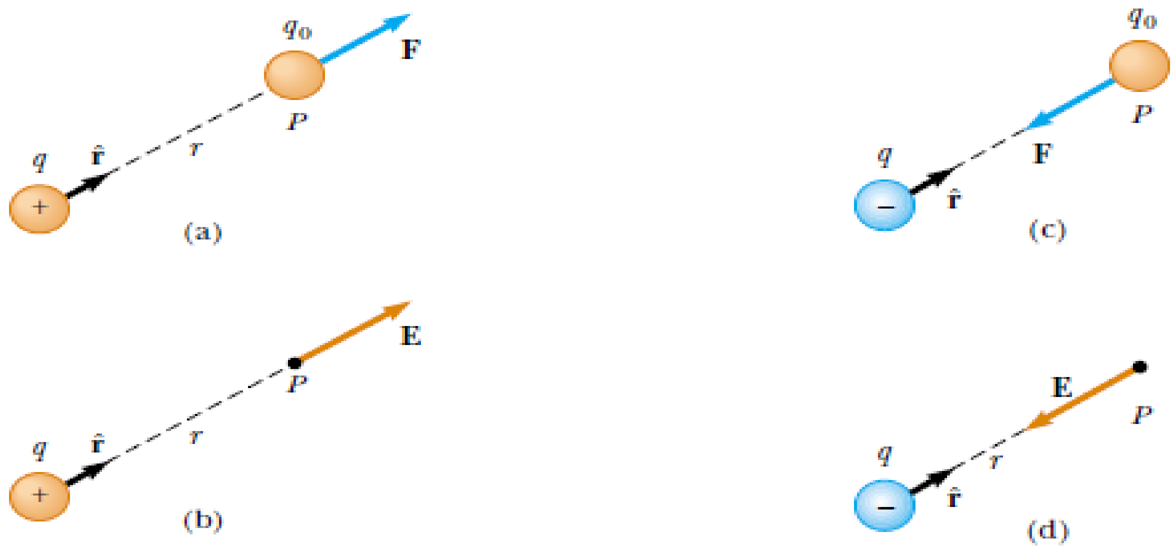
The electric field lines for a point charge +2*q* and a second point charge -*q.* Note that two lines leave +2*q* *for* every one that terminates on -*q.*



23.4 The Electric Field

$$\mathbf{F}_e = q\mathbf{E}$$

This equation gives us the force on a charged particle placed in an electric field. If q is positive, the force is in the same direction as the field. If q is negative, the force and the field are in opposite directions.

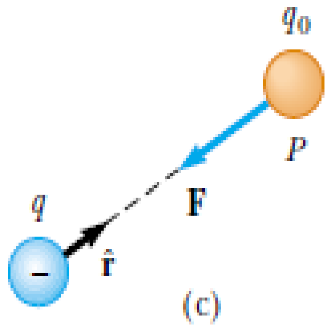


23.4 The Electric Field

$$\mathbf{F}_e = k_e \frac{q q_0}{r^2} \hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from q toward q_0. This force in Figure 23.13a is directed away from the source charge q. Because the electric field at P, the position of the test charge, is defined by $\mathbf{E} = \mathbf{F}_e/q_0$, we find that at P, the electric field created by q is

$$\mathbf{E} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$



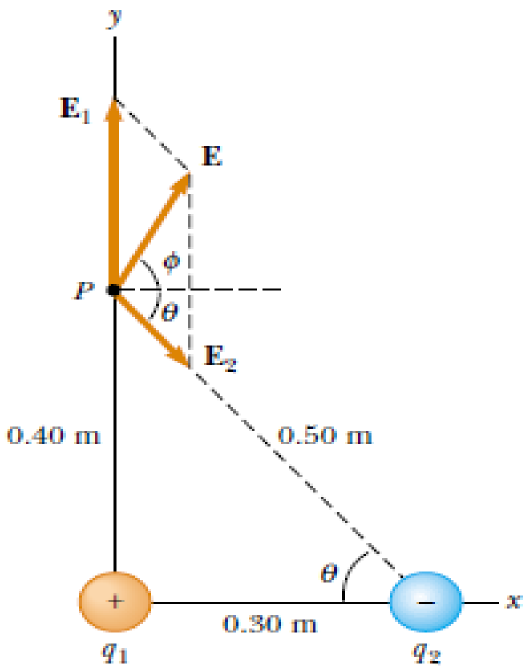
at any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

where r_i is the distance from the ith source charge q_i to the point P and $\hat{\mathbf{r}}_i$ is a unit vector directed from q_i toward P.

Example 23.5 **Electric Field Due to Two Charges**

A charge $q_1 = 7.0\ \mu\text{C}$ is located at the origin, and a second charge $q_2 = -5.0\ \mu\text{C}$ is located on the x axis, 0.30 m from the origin (Fig. 23.14). Find the electric field at the point P, which has coordinates (0, 0.40) m.



Solution First, let us find the magnitude of the electric field at P due to each charge. The fields $\mathbf{E}_1$ due to the 7.0-μC charge and $\mathbf{E}_2$ due to the -5.0-μC charge are shown in Figure 23.14. Their magnitudes are

$$E_1 = k_e \frac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(7.0 \times 10^{-6} \text{ C})}{(0.40 \text{ m})^2}$$
$$= 3.9 \times 10^5 \text{ N/C}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \frac{(5.0 \times 10^{-6} \text{ C})}{(0.50 \text{ m})^2}$$
$$= 1.8 \times 10^5 \text{ N/C}$$

The vector $\mathbf{E}_1$ has only a y component. The vector $\mathbf{E}_2$ has an x component given by $E_2 \cos\theta = \frac{3}{5}E_2$ and a negative y component given by $-E_2 \sin\theta = -\frac{4}{5}E_2$. Hence, we can express the vectors as

$$\mathbf{E}_1 = 3.9 \times 10^5 \hat{\mathbf{j}} \text{ N/C}$$

$$\mathbf{E}_2 = (1.1 \times 10^5 \hat{\mathbf{i}} - 1.4 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}$$

The resultant field $\mathbf{E}$ at P is the superposition of $\mathbf{E}_1$ and $\mathbf{E}_2$:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \boxed{(1.1 \times 10^5 \hat{\mathbf{i}} + 2.5 \times 10^5 \hat{\mathbf{j}}) \text{ N/C}}$$

From this result, we find that $\mathbf{E}$ makes an angle ϕ of 66° with the positive x axis and has a magnitude of 2.7×10^5 N/C.

Electric Field: Example 2

Calculate the magnitude and direction of an electric field at a point 30 cm from a source charge of $Q = -3.0 \times 10^{-6}$ C.

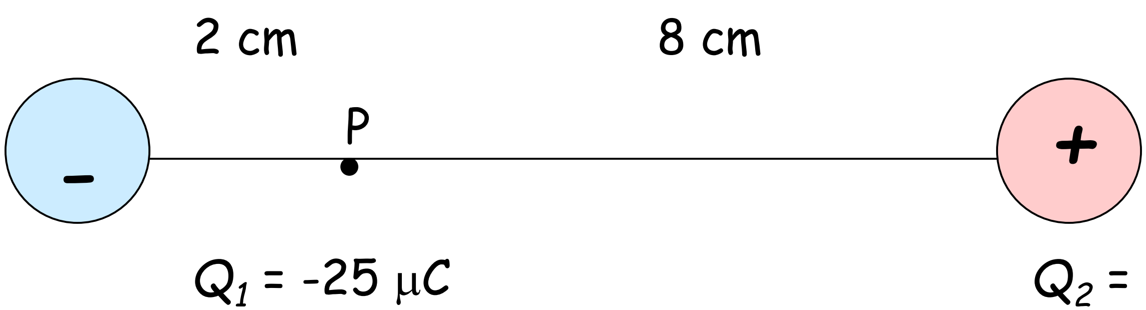
$$E = \frac{kQ}{r^2}$$

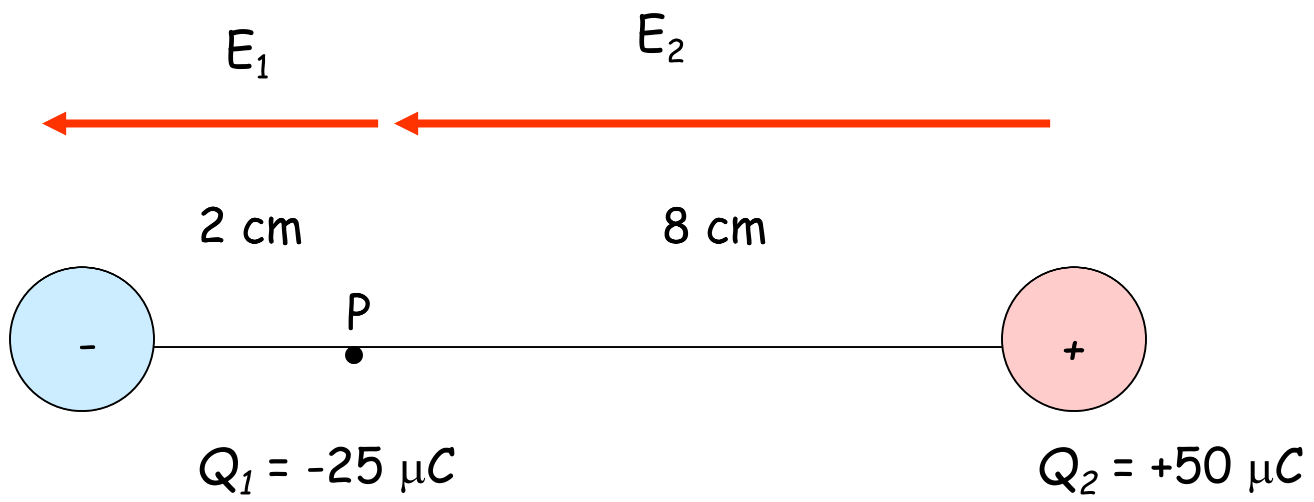
$$E = \frac{(9.0 \times 10^{9} \text{ N-m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})}{(0.30 \text{ m})^2}$$

E = 3.05 X 10⁵ N/C *towards* the charge

Electric Field: Example 3

Two point charges are separated by a distance of 10.0 cm. What is the magnitude and direction of the electric field at point P, 2.0 cm from the negative charge?




E_1
E_2
2 cm
8 cm
P
-
+
Q_1 = -25 μC
Q_2 = +50 μC

$E = E_1 + E_2$ (both point to the left)

$$E = \frac{kQ}{r^2}$$

$$E_1 = \frac{(9.0 \times 10^9 \text{ N-m}^2/\text{C}^2)(25 \times 10^{-6} \text{ C})}{(0.020 \text{ m})^2}$$

$E_1 = 5.625 \times 10^8$ N/C

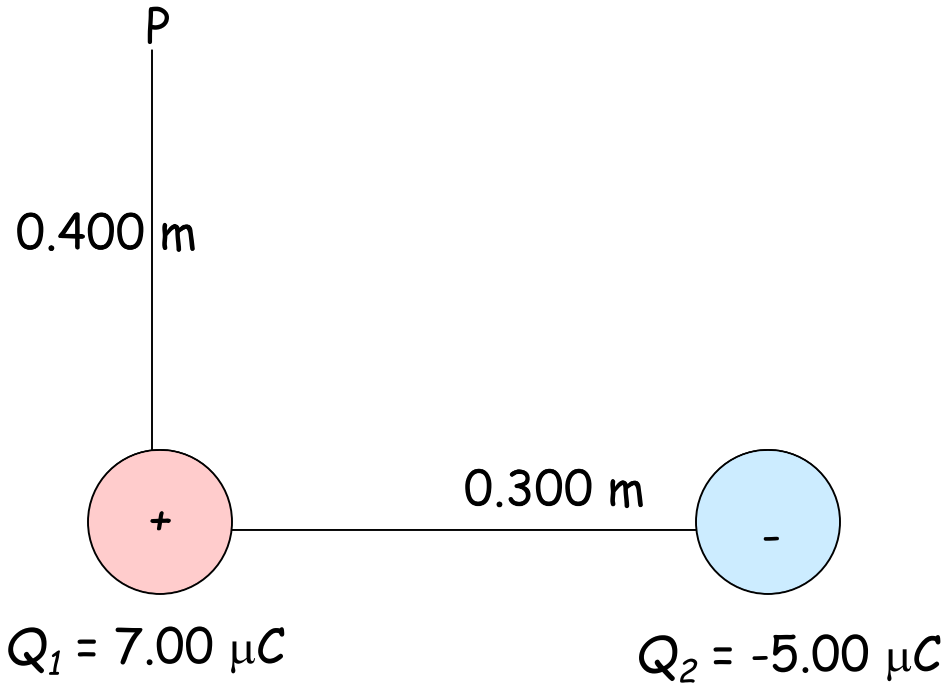
$$E_2 = \frac{(9.0 \times 10^9 \text{ N-m}^2/\text{C}^2)(50 \times 10^{-6} \text{ C})}{(0.080 \text{ m})^2}$$

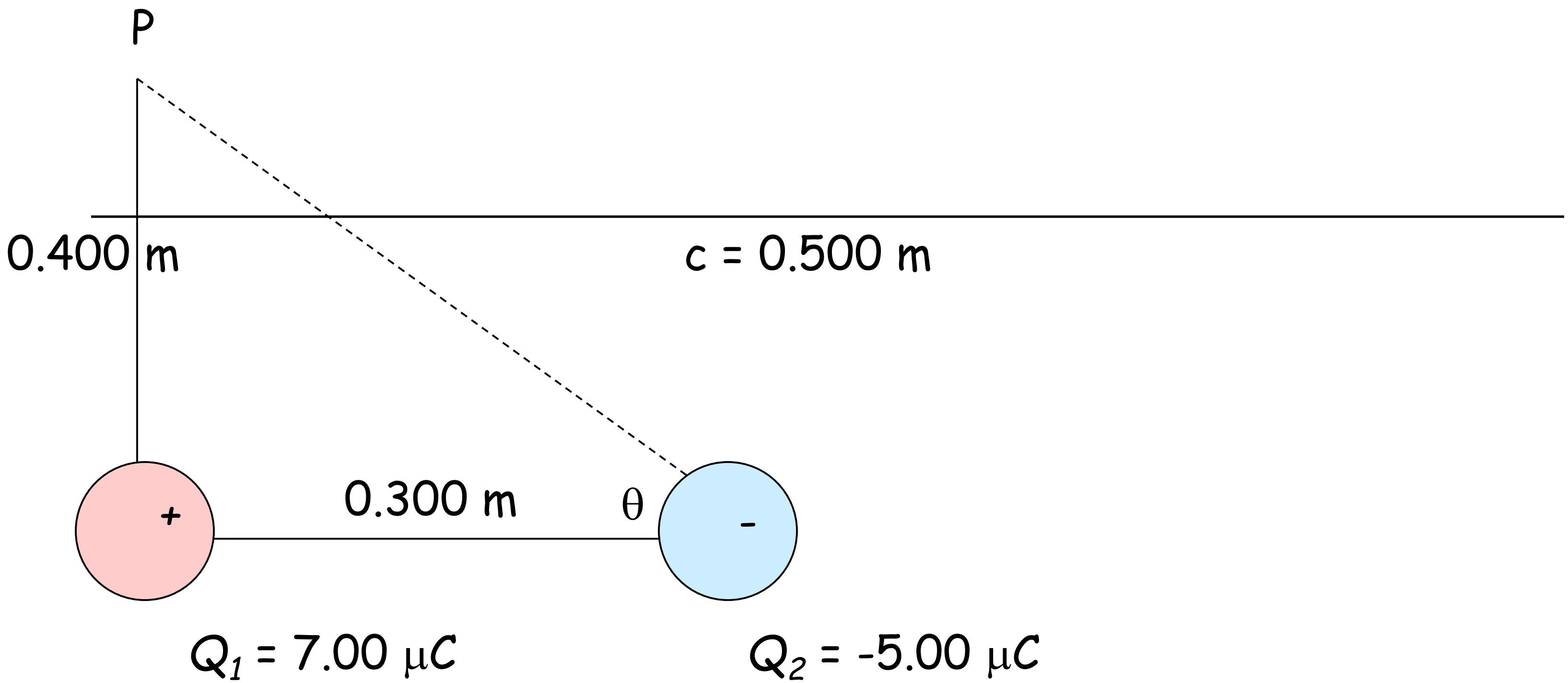
$E_2 = 7.031 \times 10^7$ N/C

$E = E_1 + E_2 = 6.3 \times 10^8$ N/C

Electric Field: Example 4

Charge Q_1 = 7.00 μC is placed at the origin. Charge Q_2 = -5.00 μC is placed 0.300 m to the right. Calculate the electric field at point P, 0.400 m above the origin.





$c^2 = a^2 + b^2$

$c^2 = (0.400\text{ m})^2 + (0.300\text{ m})^2$

$c = 0.500\text{ m}$

$\tan\theta = \text{opp}/\text{adj} = 0.400/0.300$

$\theta = 53.1^{\circ}$

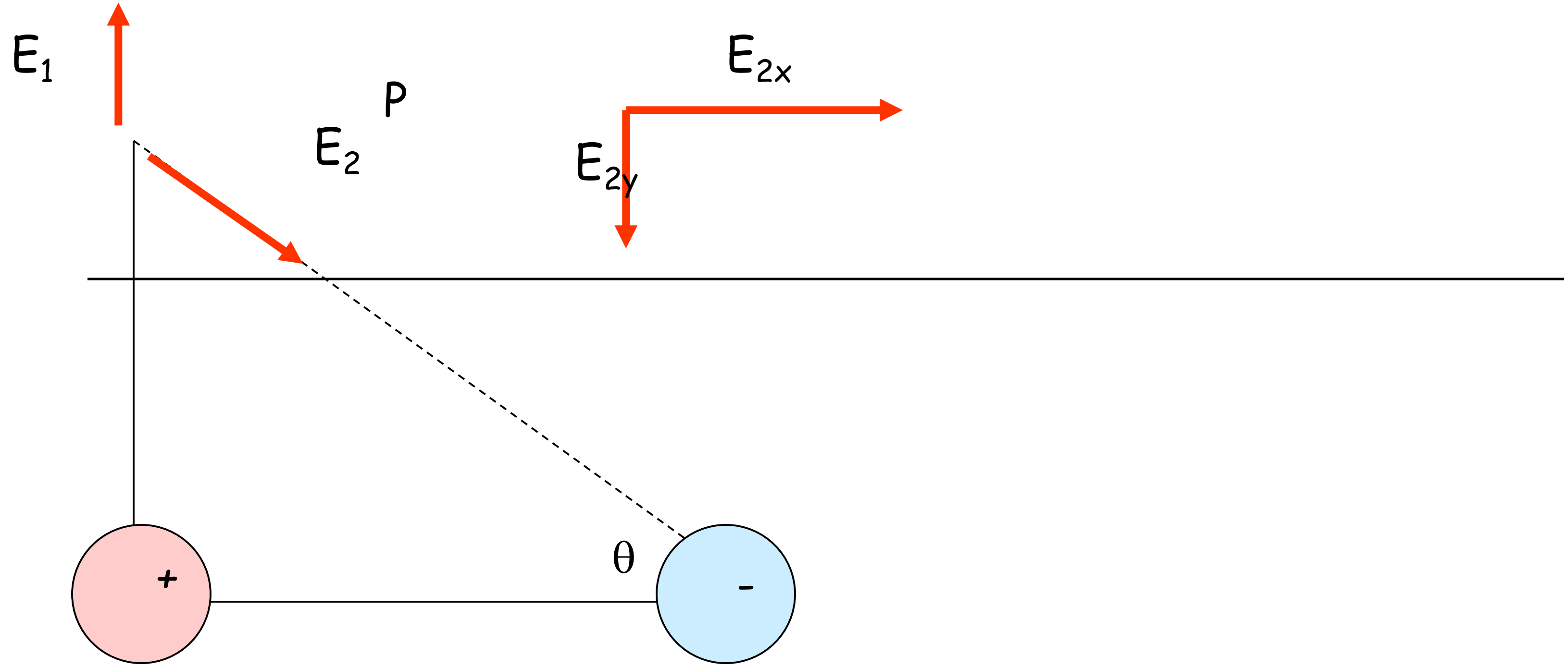
$$E = \frac{kQ}{r^2}$$

$$E_1 = \frac{(9.0 \times 10^9\ N\text{-}m^2/C^2)(7.00 \times 10^{-6}\ C)}{(0.400\ m)^2}$$

$$\mathbf{E_1 = 3.94 \times 10^5\ N/C}$$

$$E_2 = \frac{(9.0 \times 10^9\ N\text{-}m^2/C^2)(5.00 \times 10^{-6}\ C)}{(0.500\ m)^2}$$

$$\mathbf{E_2 = 1.80 \times 10^5\ N/C}$$



$$E_{2x} = E_2\cos\theta = (1.80 \times 10^5\ N/C)(\cos 53.1^\circ)$$

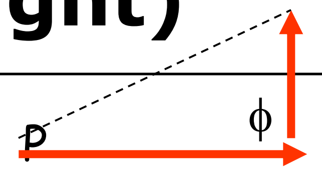
$$E_{2x} = 1.08 \times 10^5\ N/C \text{ (to the right)}$$

$$E_{2y} = E_2\sin\theta = (1.80 \times 10^5\ N/C)(\sin 53.1^\circ)$$

$$E_{2y} = -1.44 \times 10^5\ N/C \text{ (down)}$$

$E_x = E_{2x}$

E_x = **1.08×10^5 N/C (to the right)**



$E_y = E_1 + E_{2y}$

$E_y = 3.94 \times 10^5\ \mathrm{N/C} + -1.44 \times 10^5\ \mathrm{N/C}$

$E_y = 2.49 \times 10^5$ N/C

$E_R^2 = (1.08 \times 10^5\ \mathrm{N/C})^2 + (2.49 \times 10^5\ \mathrm{N/C})^2$

$E_R = 2.72 \times 10^5$ N/C

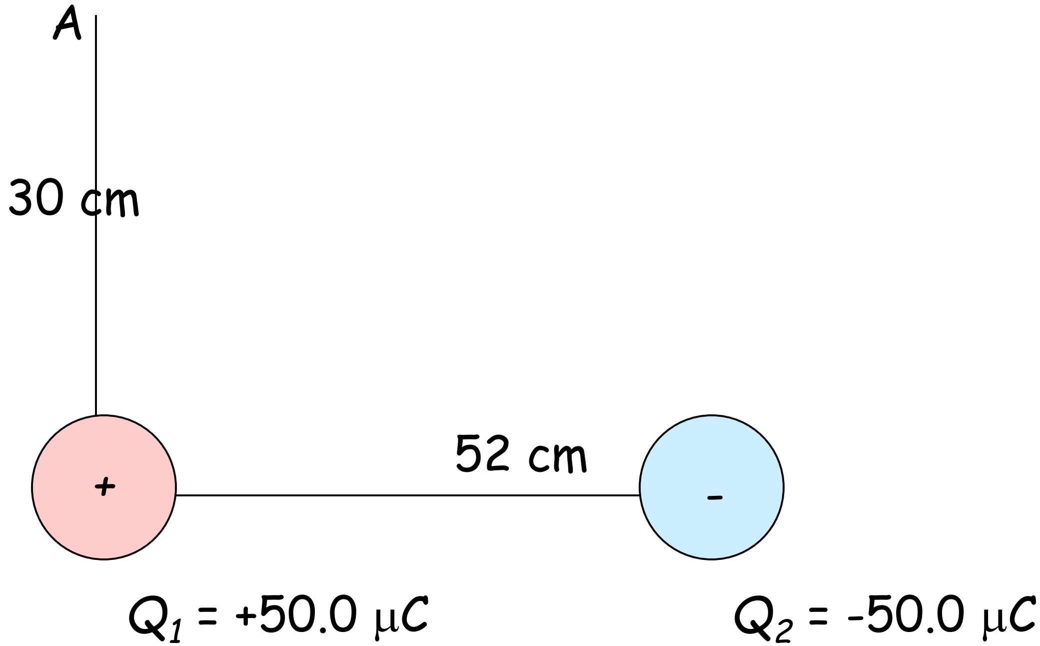
$\tan \phi = E_y/E_x = 2.49 \times 10^5 / 1.08 \times 10^5$

$\phi = 66.6^o$

Electric Field: Example 5

Calculate the electric field at point A, as shown in the diagram

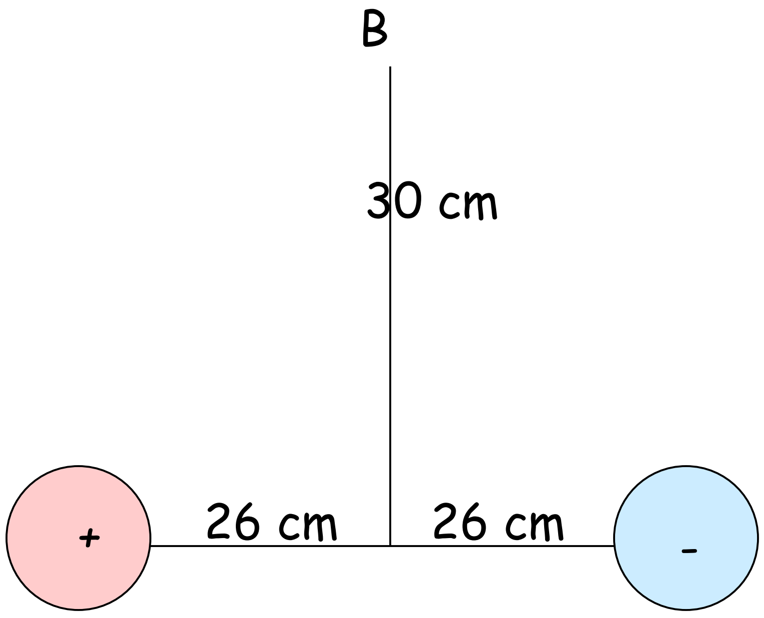
Ans: E = 4.5 X 10^6 N/C at an angle of 76^o



Electric Field: Example 6

Calculate the electric field at point B, as shown in the diagram.

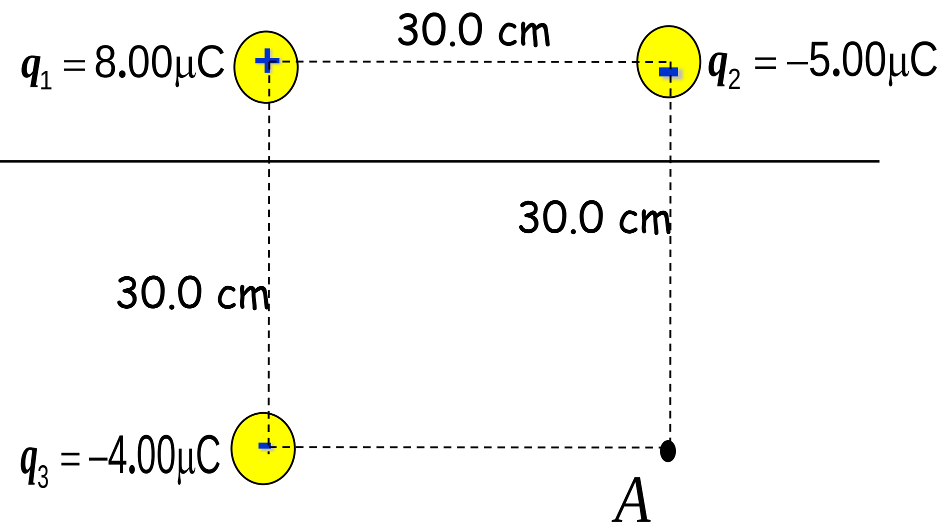
Ans: $E = 3.6 \times 10^6$ N/C along the +x direction



$Q_1 = +50.0\ \mu C$ $Q_2 = -50.0\ \mu C$

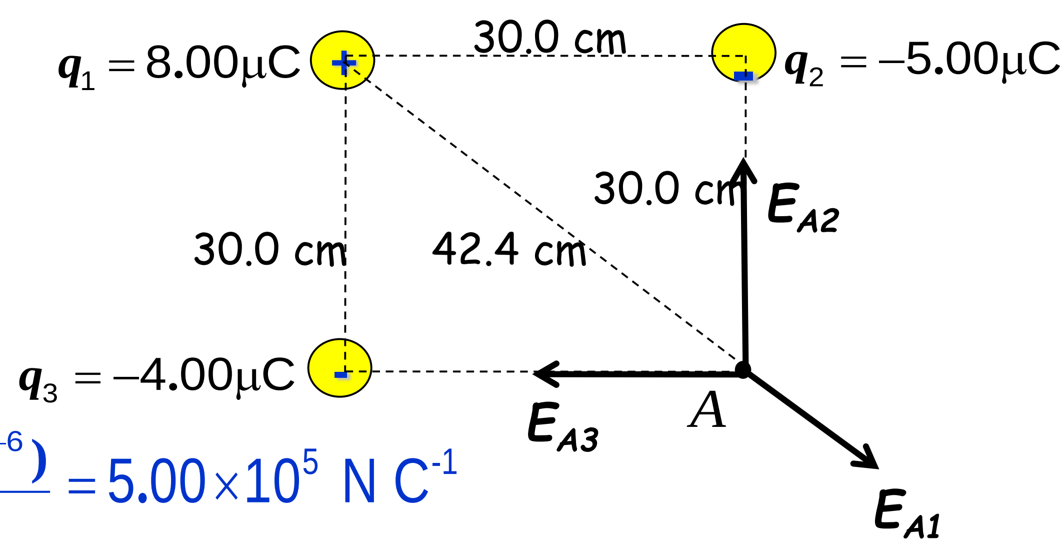
Example

Three charges are placed on three corners of a square, as shown above. Each side of the square is 30.0 cm. Calculate the electric field strength at point A. What would be the force on a 6.00 μC charge placed at the point A?



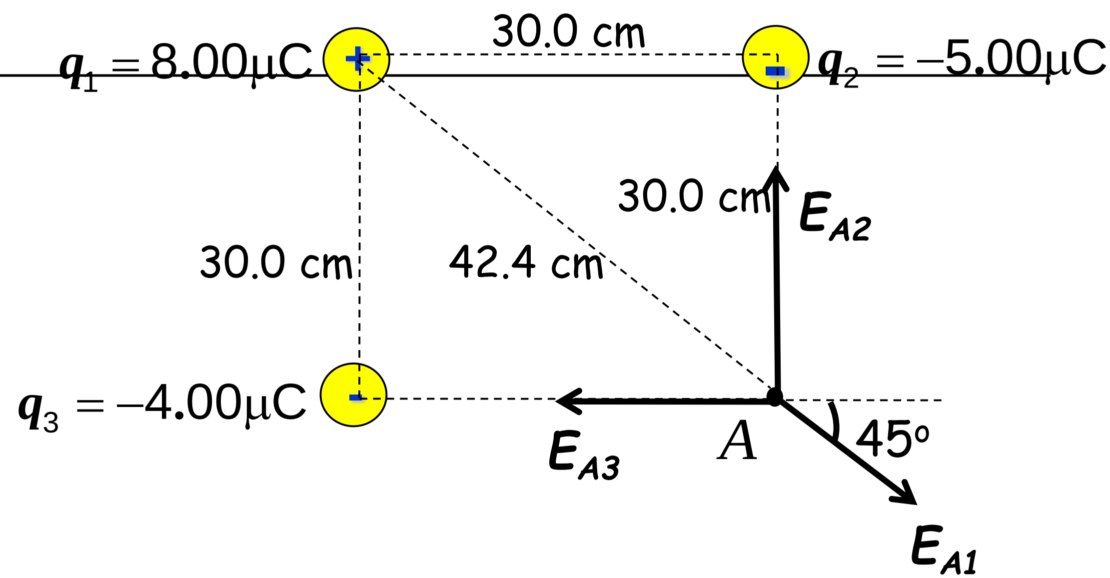
$$E_{A1} = \frac{kq_1}{r_1^2} = \frac{(9.0\times10^9)(8.00\times10^{-6})}{(42.4\times10^{-2})^2}$$

$$= 4.00\times10^5 \text{ N C}^{-1}$$



$$E_{A2} = \frac{kq_2}{r_2^2} = \frac{(9.0\times10^9)(5.00\times10^{-6})}{(30.0\times10^{-2})^2} = 5.00\times10^5 \text{ N C}^{-1}$$

$$E_{A3} = \frac{kq_3}{r_3^2} = \frac{(9.0\times10^{9})(4.00\times10^{-6})}{(30.0\times10^{-2})^2} = 4.00\times10^{5}\ \text{N C}^{-1}$$



$$\sum E_{AX} = E_{A1}\cos 45^{o} - E_{A3}$$

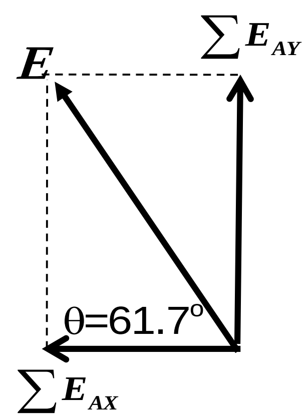
$$= -1.17\times10^{5}\text{N/C}$$

$$\sum E_{AY} = E_{A2} - E_{A1}\sin 45^{o}$$

$$= 2.17\times10^{5}\text{N/C}$$

$$E = \sqrt{\sum E_{AX}{}^{2} + \sum E_{AY}{}^{2}}$$

$$E = 2.46\times10^{5}\ \text{N/C}$$



$$\tan\theta = \frac{\sum E_{AY}}{\sum E_{AX}}$$

$$\theta = 61.7^{o}$$

23.6 Electric Field Lines

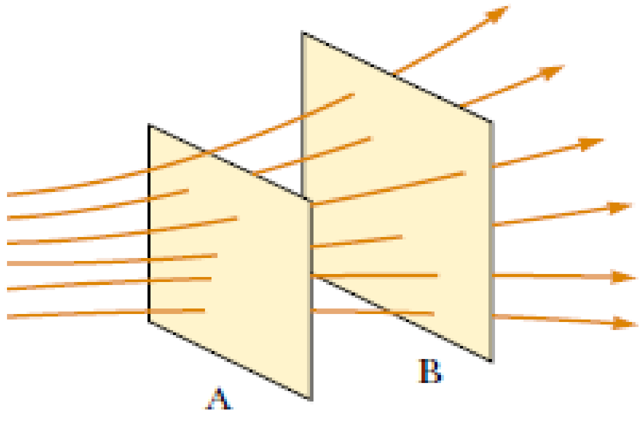
23.6 Electric Field Lines

A convenient way of visualizing electric field patterns is to draw curved lines that are parallel to the electric field vector at any point in space. These lines, called *electric field lines* and first introduced by Faraday, are related to the electric field in a region of space in the following manner:

- The electric field vector E is tangent to the electric field line at each point. The line has a direction, indicated by an arrowhead, that is the same as that of the electric field vector.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region. Thus, the field lines are close together where the electric field is strong and far apart where the field is weak.

23.6 Electric Field Lines

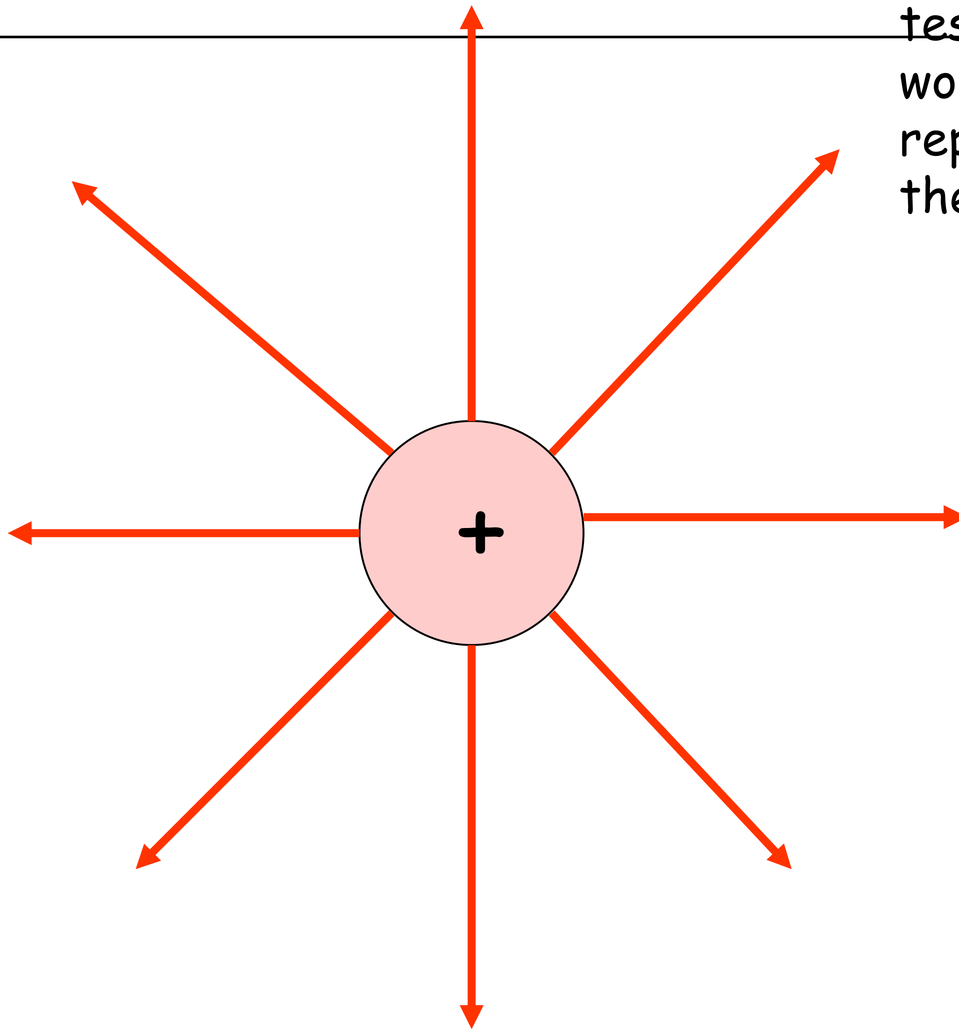
These properties are illustrated in Figure. The density of lines through surface A is greater than the density of lines through surface B. Therefore, the magnitude of the electric field is larger on surface A than on surface B. Furthermore, the fact that the lines at different locations point in different directions indicates that the field is nonuniform.



23.6 Electric Field Lines

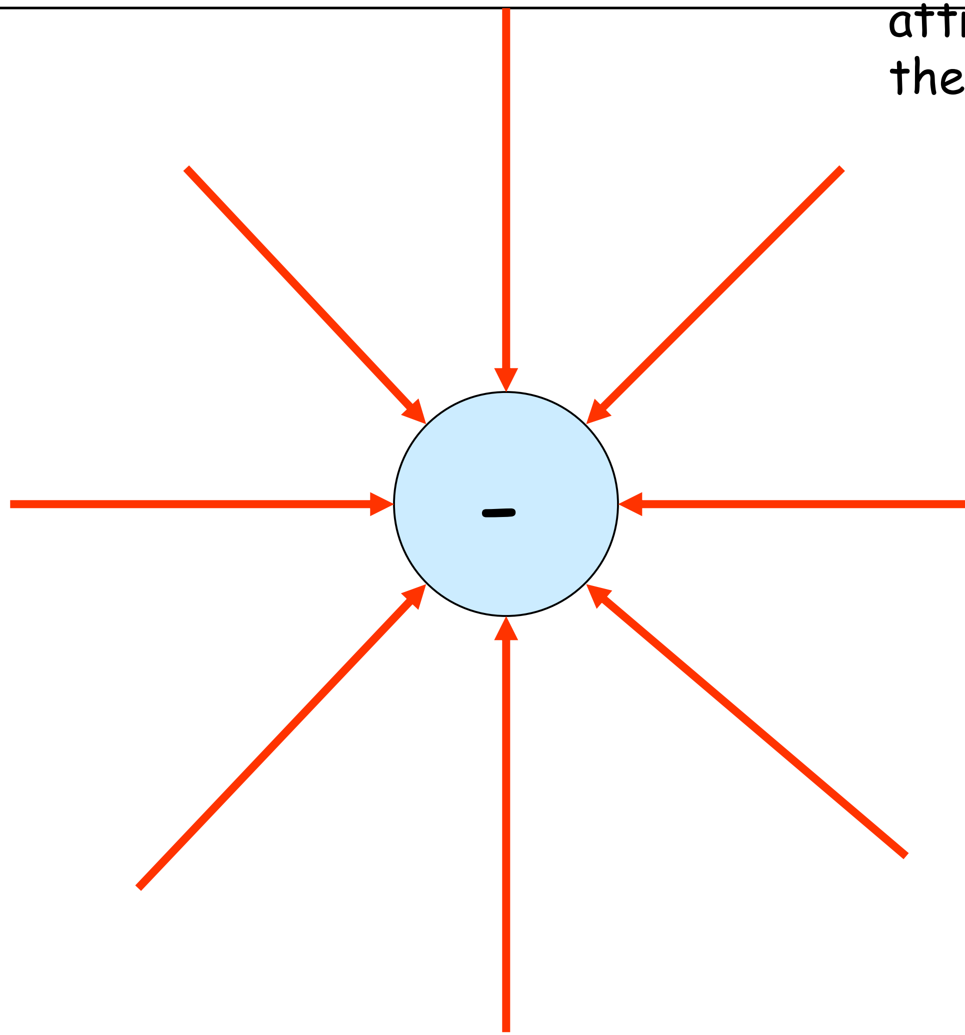
Proton

- A positive test charge would be repelled by the field

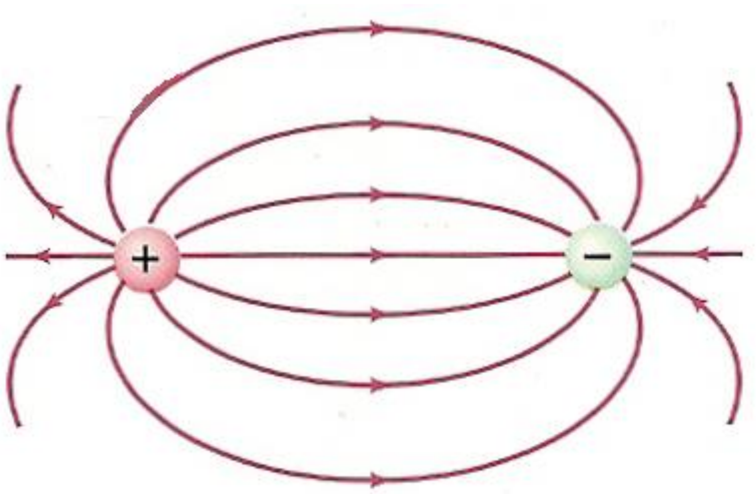


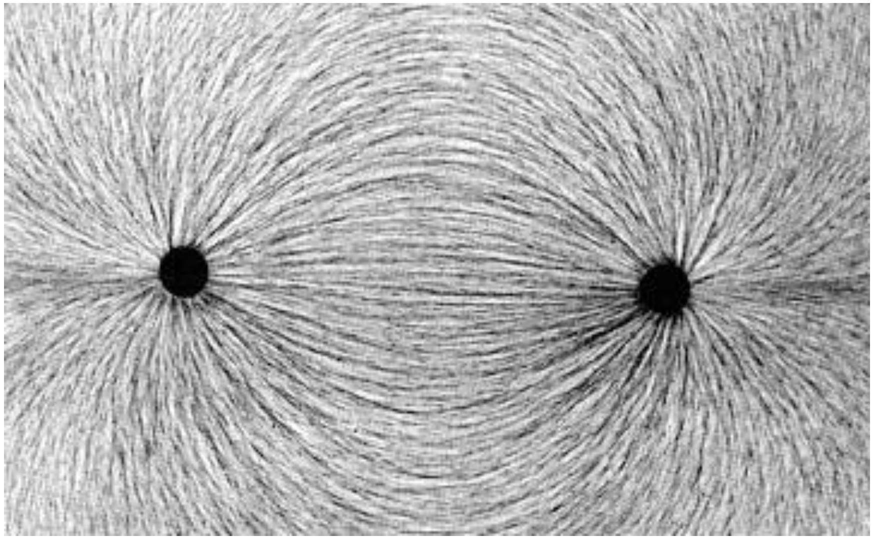
Electron

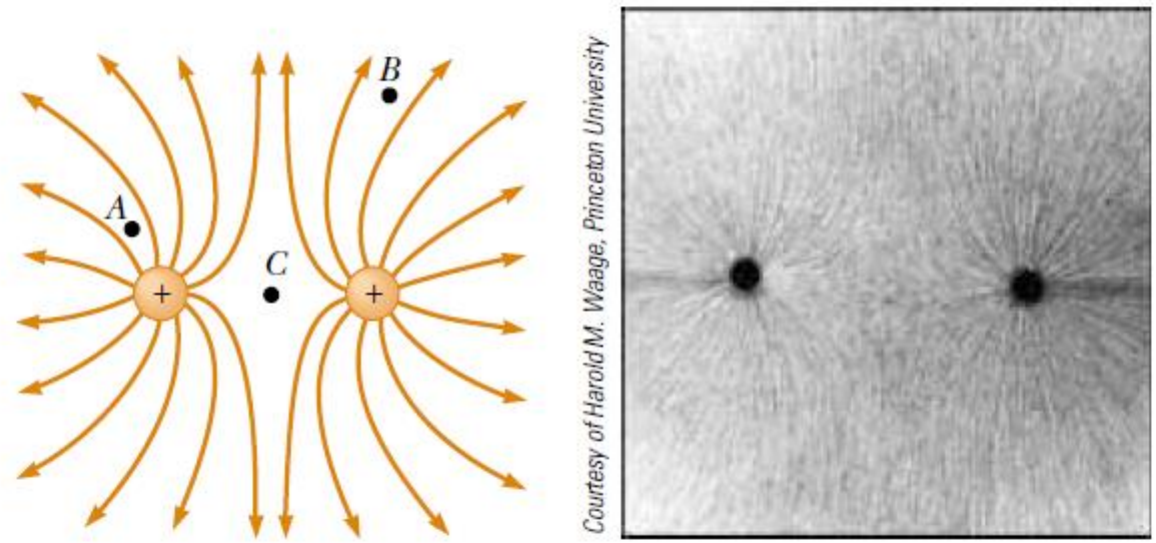
A positive test charge + would be attracted by the field



Opposite charges attract



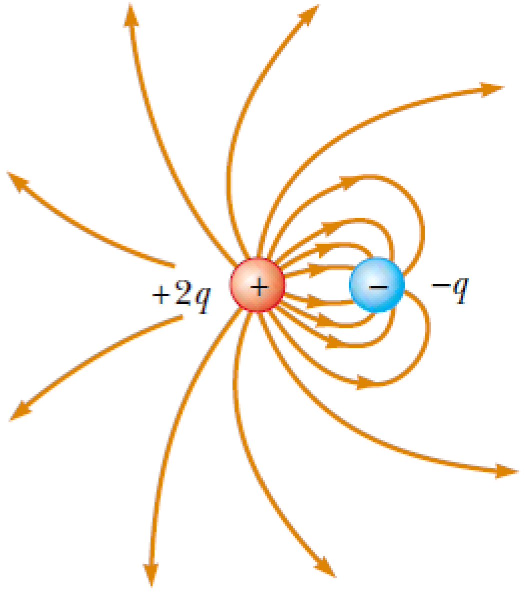




(a) (b)

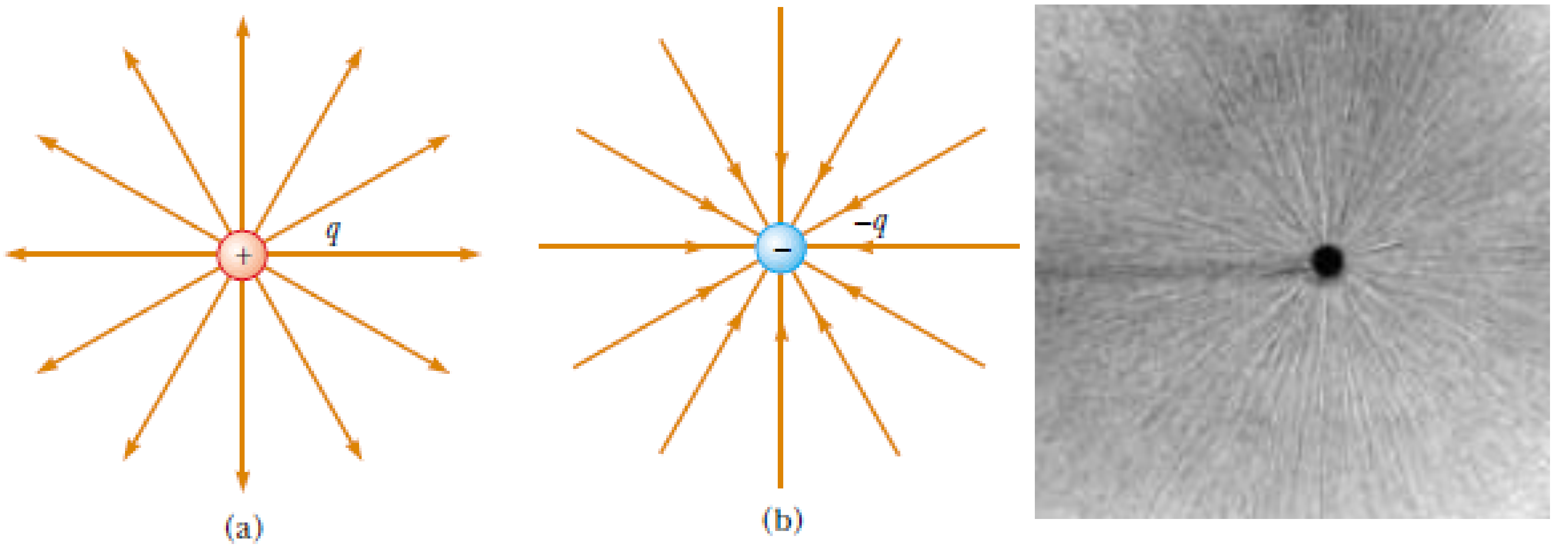
(a) The electric field lines for two positive point charges. (The locations *A, B, and C are discussed in Quick Quiz 23.7.) (b) Pieces of thread suspended in oil,* which align with the electric field created by two equal-magnitude positive charges.

The electric field lines for a point charge $+2q$ and a second point charge $-q$. Note that two lines leave $+2q$ *for* every one that terminates on $-q$.



23.6 Electric Field Lines

Representative electric field lines for the field due to a single point charge:



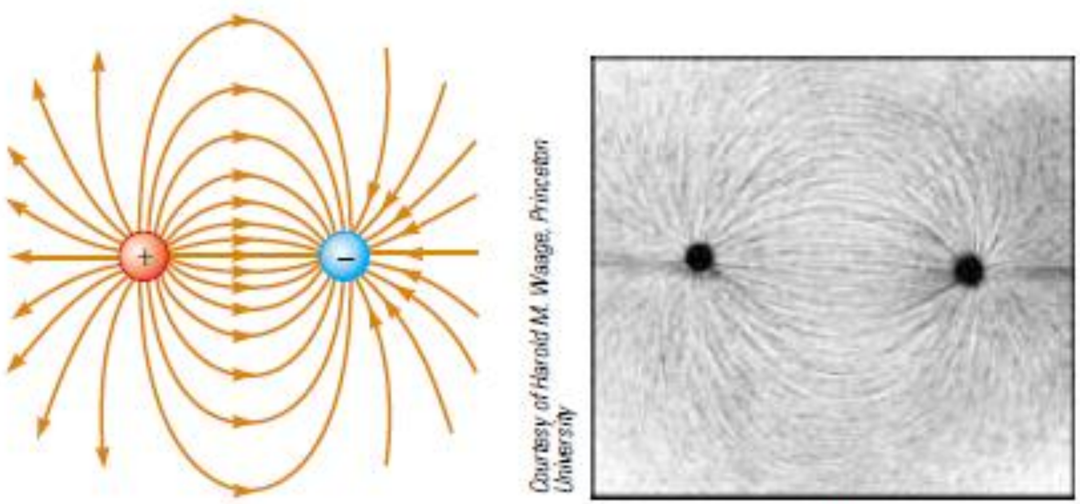
23.6 Electric Field Lines

The rules for drawing electric field lines are as follows:

- The lines must begin on a positive charge and terminate on a negative charge. In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross.

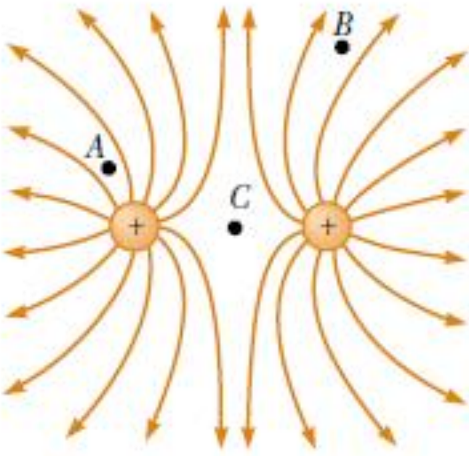
23.6 Electric Field Lines

We choose the number of field lines starting from any positively charged object to be Cq and the number of lines ending on any negatively charged object to be $C|q|$, where C is an arbitrary proportionality constant. Once C is chosen, the number of lines is fixed. For example, if object 1 has charge Q_1 and object 2 has charge Q_2, then the ratio of number of lines is $N_2/N_1 = Q_2/Q_1$. The electric field lines for two point charges of equal magnitude but opposite signs (an electric dipole) are shown in Figure 23.22. Because the charges are of equal magnitude, the number of lines that begin at the positive charge must equal the number that terminate at the negative charge. At points very near the charges, the lines are nearly radial. The high density of lines between the charges indicates a region of strong electric field.

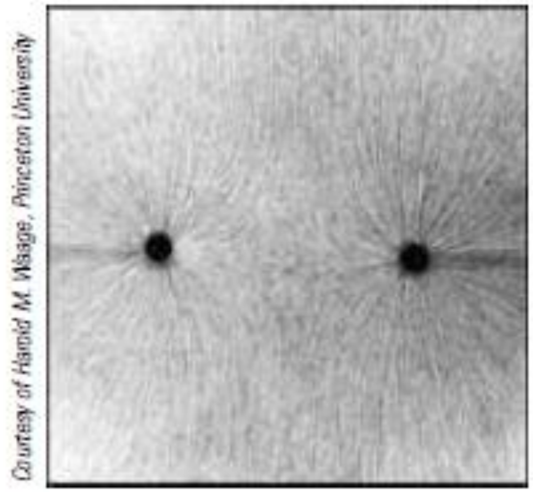


(a) (b)

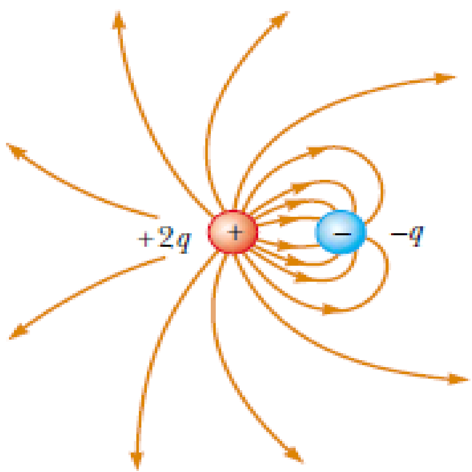
23.6 Electric Field Lines



(a)



(b)



SUMMARY

Electric charges have the following important properties:

- Charges of opposite sign attract one another and charges of the same sign repel one another.
- Total charge in an isolated system is conserved.
- Charge is quantized.

Coulomb's law states that the electric force exerted by a charge q_1 on a second charge q_2 is

$$\mathbf{F}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}$$

where r is the distance between the two charges and $\hat{\mathbf{r}}$ is a unit vector directed from q_1 toward q_2. The constant k_e, which is called the Coulomb constant, has the value $k_e = 8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$.

The smallest unit of free charge e known to exist in nature is the charge on an electron ($-e$) or proton ($+e$), where $e = 1.602\,19 \times 10^{-19}$ C.

The electric field $\mathbf{E}$ at some point in space is defined as the electric force $\mathbf{F}_e$ that acts on a small positive test charge placed at that point divided by the magnitude q_0 of the test charge:

$$\mathbf{E} \equiv \frac{\mathbf{F}_e}{q_0}$$

Thus, the electric force on a charge q placed in an electric field $\mathbf{E}$ is given by

$$\mathbf{F}_e = q\mathbf{E}$$

At a distance r from a point charge q, the electric field due to the charge is given by

$$\mathbf{E} = k_e \frac{q}{r^2}\,\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is a unit vector directed from the charge toward the point in question. The electric field is directed radially outward from a positive charge and radially inward toward a negative charge.

The electric field due to a group of point charges can be obtained by using the superposition principle. That is, the total electric field at some point equals the vector sum of the electric fields of all the charges:

$$\mathbf{E} = k_e \sum_i \frac{q_i}{r_i^{\,2}} \hat{\mathbf{r}}_i$$

Electric field lines describe an electric field in any region of space. The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of **E** in that region.