Chapter 26

Capacitance and Dielectrics

CHAPTER OUTLINE

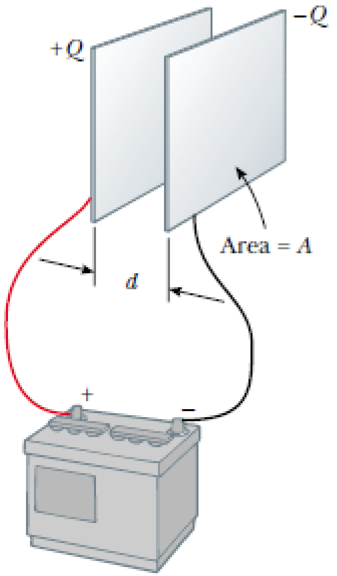
26.1 Definition of Capacitance
26.2 Calculating Capacitance
26.3 Combinations of Capacitors
26.4 Energy Stored in a Charged Capacitor
26.5 Capacitors with Dielectrics

26.1 Definition of Capacitance

The capacitance C *of a capacitor is defined as the ratio of the magnitude of the* charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V}$$

Note that by definition *capacitance is always a positive quantity. Furthermore, the charge* Q *and the potential difference* ΔV *are positive* quantities. Because the potential difference increases linearly with the stored charge, the ratio $Q / \Delta V$ *is constant for a given capacitor*.



The SI unit of capacitance is the farad (F),

$$1\ \mathrm{F} = 1\ \mathrm{C/V}$$

Capacitance

- The charge, Q, on a capacitor is directly proportional to the potential difference, V, across the capacitor. That is,

  $$Q \propto V$$

- Introducing a constant, C, known as the capacitance of the capacitor, we have

  $$Q = CV$$

- Capacitance of a capacitor is defined as the ratio of charge on one of the capacitor plates to the potential difference between the plates.

- Charge Q is measured in coulombs, C.
- Potential difference, V, is measured in volts, V.
- Capacitance, C, is measured in farads, F.
- 1 farad is 1 coulomb per volt: $1\ \text{F} = 1\ \text{C V}^{-1}$

- 1 farad is a very large unit. It is much more common to use the following:

  $\text{mF} = 10^{-3}\ \text{F}$

  $\mu\text{F} = 10^{-6}\ \text{F}$

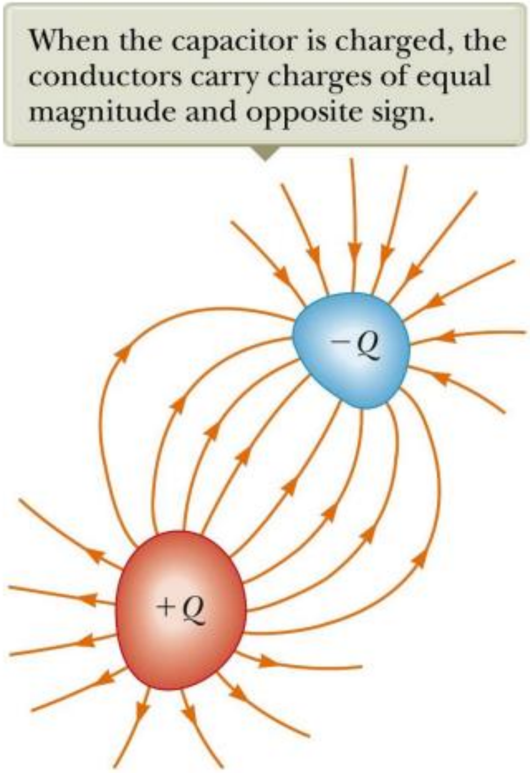
  $\text{nF} = 10^{-9}\ \text{F}$

  $\text{pF} = 10^{-12}\ \text{F}$

- Capacitors are devices that store electric charge.
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

Makeup of a Capacitor

- A capacitor consists of two conductors.
  - These conductors are called plates.
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions.
- A potential difference exists between the plates due to the charge.



Capacitance:

To store charge & To store energy

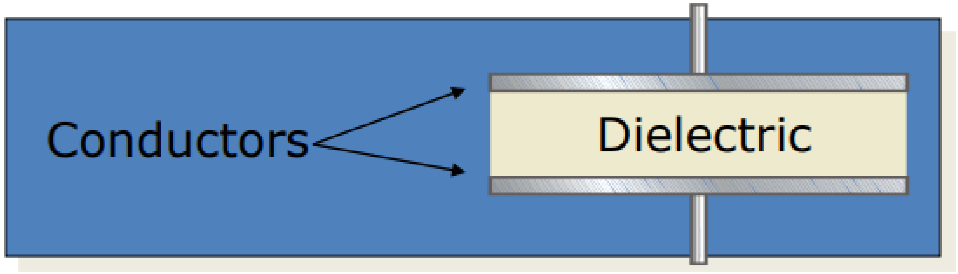
To control variation time scales in a circuit

The Capacitor

Capacitors are one of the fundamental passive components.

In its most basic form, it is composed of two plates separated by a dielectric.

The ability to store charge is the definition of **capacitance**.



Parallel-Plate Capacitors

Two parallel metallic plates of equal area *A are separated by a distance d*, One plate carries a charge *Q , and the other carries a charge -Q* .

The value of the electric field between the plates is

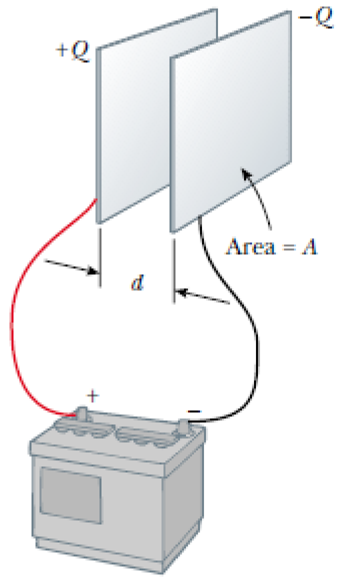
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Because the field between the plates is uniform, the magnitude of the potential difference

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d}$$



That is, the capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation

Example 26.1 **Parallel-Plate Capacitor**

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ \mathrm{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(2.00 \times 10^{-4}\ \mathrm{m^2})}{1.00 \times 10^{-3}\ \mathrm{m}}$$

$$= 1.77 \times 10^{-12}\ \mathrm{F} = 1.77\ \mathrm{pF}$$

26.3 Combinations of Capacitors

Parallel Combination

• The individual potential differences across capacitors connected in parallel are the same and are equal to the potential difference applied across the combination.

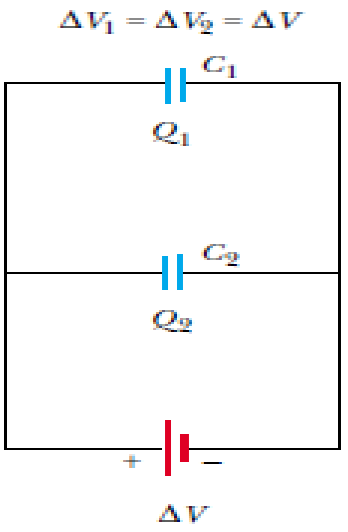
• The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors

$$Q = Q_1 + Q_2$$

$$Q_1 = C_1 \Delta V \qquad Q_2 = C_2 \Delta V$$

for the equivalent capacitor

$$Q = C_{eq} \Delta V$$



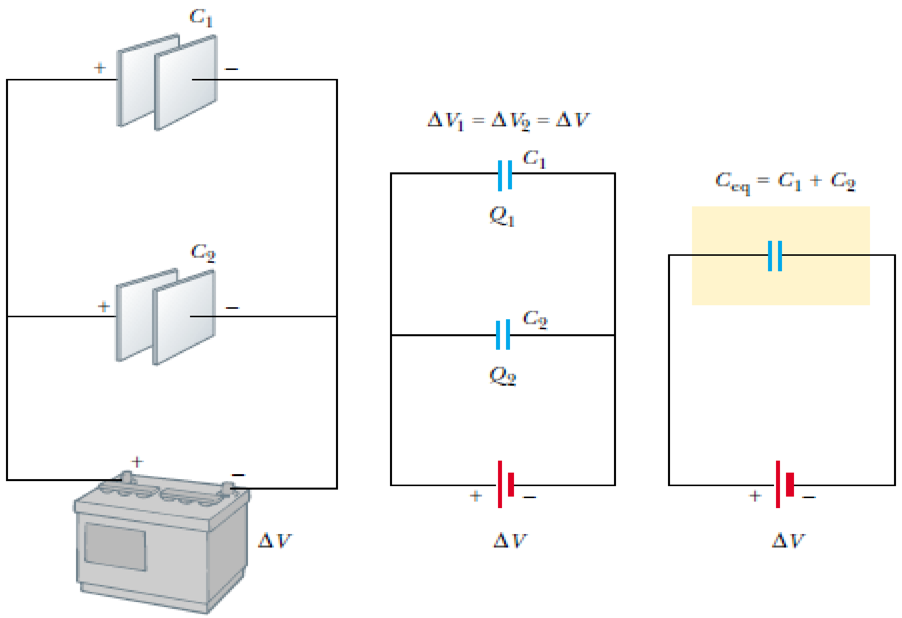
$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$

If we extend this treatment to three or more capacitors connected in parallel, we find the equivalent capacitance to be

$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

Thus, the equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances and is greater than any of the individual capacitances.



Series Combination

• The charges on capacitors connected in series are the same.

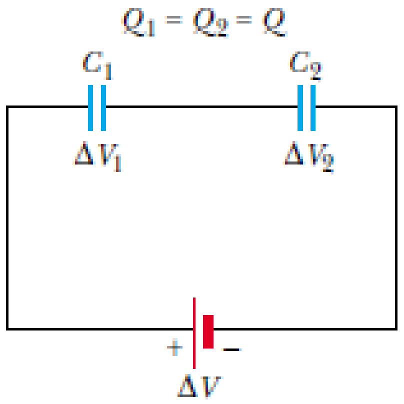
• The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = \frac{Q}{C_{eq}}$$

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

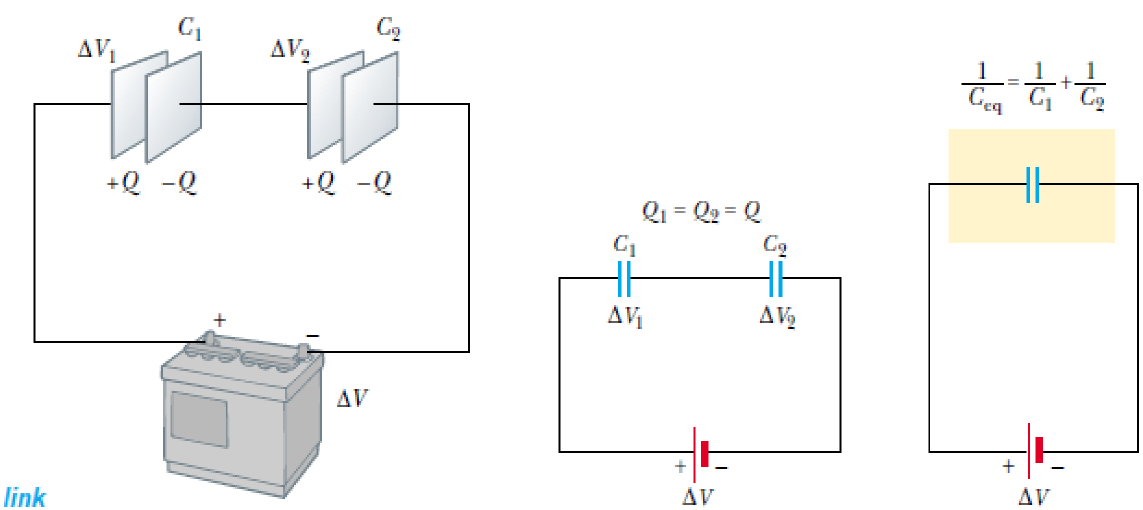


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \text{(series combination)}$$

When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(series combination)}$$

the inverse of the equivalent capacitance is the algebraic sum of the inverses of the individual capacitances and the equivalent capacitance of a series combination is always less than any individual capacitance in the combination.



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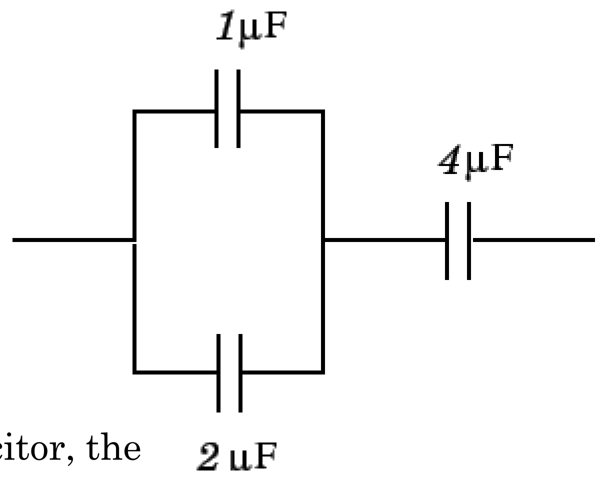
PROBLEM-SOLVING HINTS

Capacitors

- Be careful with units. When you calculate capacitance in farads, make sure that distances are expressed in meters. When checking consistency of units, remember that the unit for electric fields can be either N/C or V/m.
- When two or more capacitors are connected in parallel, the potential difference across each is the same. The charge on each capacitor is proportional to its capacitance; hence, the capacitances can be added directly to give the equivalent capacitance of the parallel combination. The equivalent capacitance is always larger than the individual capacitances.
- When two or more capacitors are connected in series, they carry the same charge, and the sum of the potential differences equals the total potential difference applied to the combination. The sum of the reciprocals of the capacitances equals the reciprocal of the equivalent capacitance, which is always less than the capacitance of the smallest individual capacitor.

Question: A $1\,\mu\text{F}$ and a $2\,\mu\text{F}$ capacitor are connected in parallel, and this pair of capacitors is then connected in series with a $4\,\mu\text{F}$ capacitor, as shown in the diagram. What is the equivalent capacitance of the whole combination? What is the charge on the $4\,\mu\text{F}$ capacitor if the whole combination is connected across the terminals of a 6V battery? Likewise, what are the charges on the $1\,\mu\text{F}$ and $2\,\mu\text{F}$ capacitors?

Solution:



The equivalent capacitance of the $1\,\mu\text{F}$ and $2\,\mu\text{F}$ capacitors connected in parallel is

$$1 + 2 = 3\,\mu\text{F}$$

When a $3\,\mu\text{F}$ capacitor is combined in series with a $4\,\mu\text{F}$ capacitor, the equivalent capacitance of the whole combination is given by

$$\frac{1}{C_{eq}} = \frac{1}{(3 \times 10^{-6})} + \frac{1}{(4 \times 10^{-6})} = \frac{(7)}{(12 \times 10^{-6})}\ \text{F}^{-1},$$

and so

$$C_{eq} = \frac{(12 \times 10^{-6})}{(7)} = 1.71\,\mu\text{F}.$$

The charge delivered by the 6 V battery is

$$Q = C_{eq}V = (1.71 \times 10^{-6})\,(6) = 10.3\,\mu\text{C}.$$

This is the charge on the $4\,\mu$F capacitor, since one of the terminals of the battery is connected directly to one of the plates of this capacitor.

The voltage drop across the $4\,\mu$F capacitor is

$$V_4 = \frac{Q}{C_4} = \frac{(10.3 \times 10^{-6})}{(4 \times 10^{-6})} = 2.57\,\text{V}.$$

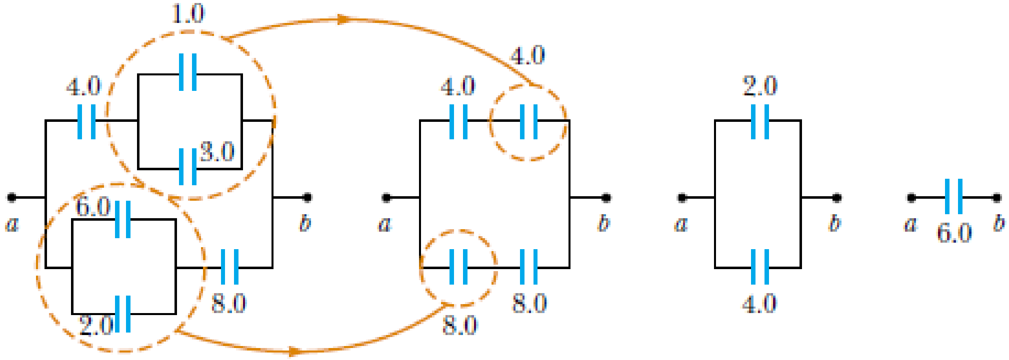
Thus, the voltage drop across the $1\,\mu$F and $2\,\mu$F combination must be $V_{12} = 6 - 2.57 = 3.43\,\text{V}$. The charge stored on the $1\,\mu$F is given by

$$Q_1 = C_1 V_{12} = (1 \times 10^{-6})\,(3.43) = 3.42\,\mu\text{C}.$$

Note that the total charge stored on the $1\,\mu\mathrm{F}$ and $2\,\mu\mathrm{F}$ combination is $Q_{12} = Q_1 + Q_2 = 10.3\,\mu\mathrm{C}$, which is the same as the charge stored on the $4\,\mu\mathrm{F}$ capacitor. This makes sense because the $1\,\mu\mathrm{F}$ and $2\,\mu\mathrm{F}$ combination and the $4\,\mu\mathrm{F}$ capacitor are connected in series.

Example 26.4 **Equivalent Capacitance**

Find the equivalent capacitance between *a and b for the* combination of capacitors shown in Figure 26.11a. All capacitances are in microfarads.



Capacitance – Isolated Sphere

- To find the capacitance of the conductor, we first rewrite the capacitance as:

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}.$$

- Now letting b→∞, and substituting *R* for *a*,

$$C = 4\pi\varepsilon_0 R \qquad \text{(isolated sphere).}$$

- Or Assume *V* = 0 for the infinitely large shell

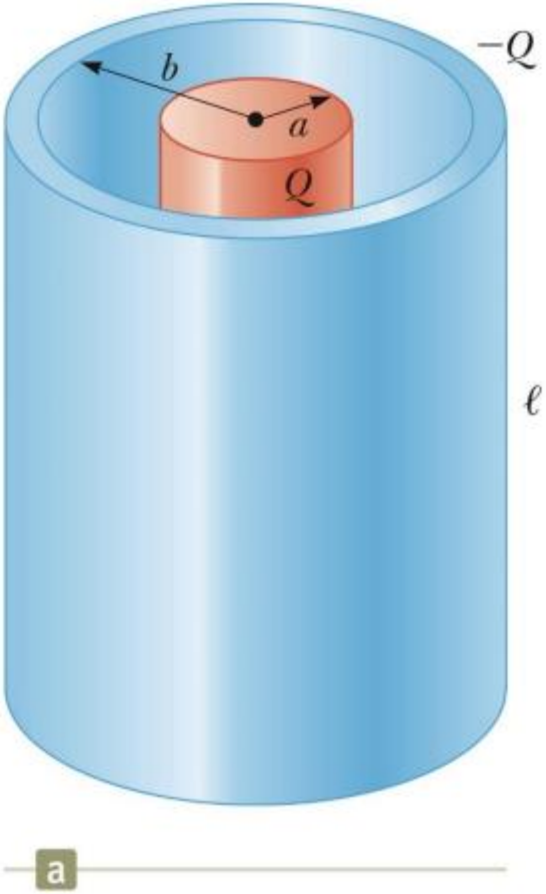
$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / a} = \frac{R}{k_e} = 4\pi\varepsilon_o a$$

- Note, this is independent of the charge on the sphere and its potential.

The Cylindrical Capacitor

- $\Delta V = -2k_e\lambda \ln(b/a)$
- $\lambda = Q/\ell$
- The capacitance is

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k_e \ln(b/a)}$$



Calculating the Capacitance; A Cylindrical Capacitor :

As a Gaussian surface, we choose a cylinder of length L and radius r, closed by end caps and placed as is shown. It is coaxial with the cylinders and encloses the central cylinder and thus also the charge q on that cylinder.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

$$EA = E(2\pi r L) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2\pi\varepsilon_0 L r}.$$

$$V = \int_{-}^{+} E\, ds = -\frac{q}{2\pi\varepsilon_0 L} \int_{b}^{a} \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right),$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)} \qquad \text{(cylindrical capacitor).}$$

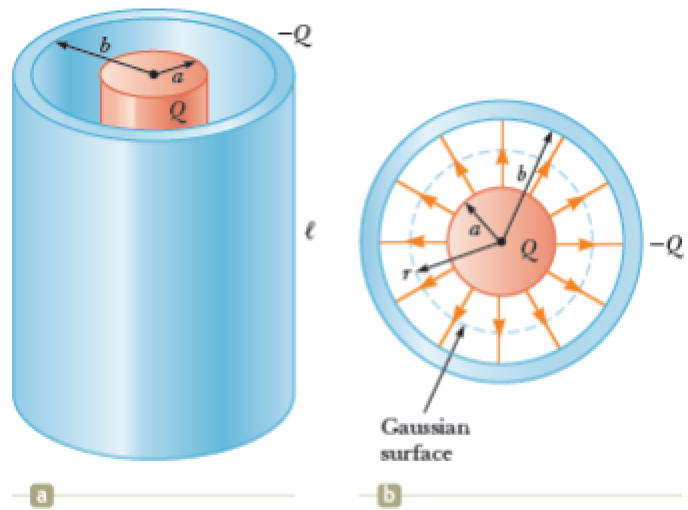


Figure 26.4 (Example 26.1) (a) A cylindrical capacitor consists of a solid cylindrical conductor of radius a and length ℓ surrounded by a coaxial cylindrical shell of radius b. (b) End view. The electric field lines are radial. The dashed line represents the end of a cylindrical gaussian surface of radius r and length ℓ.

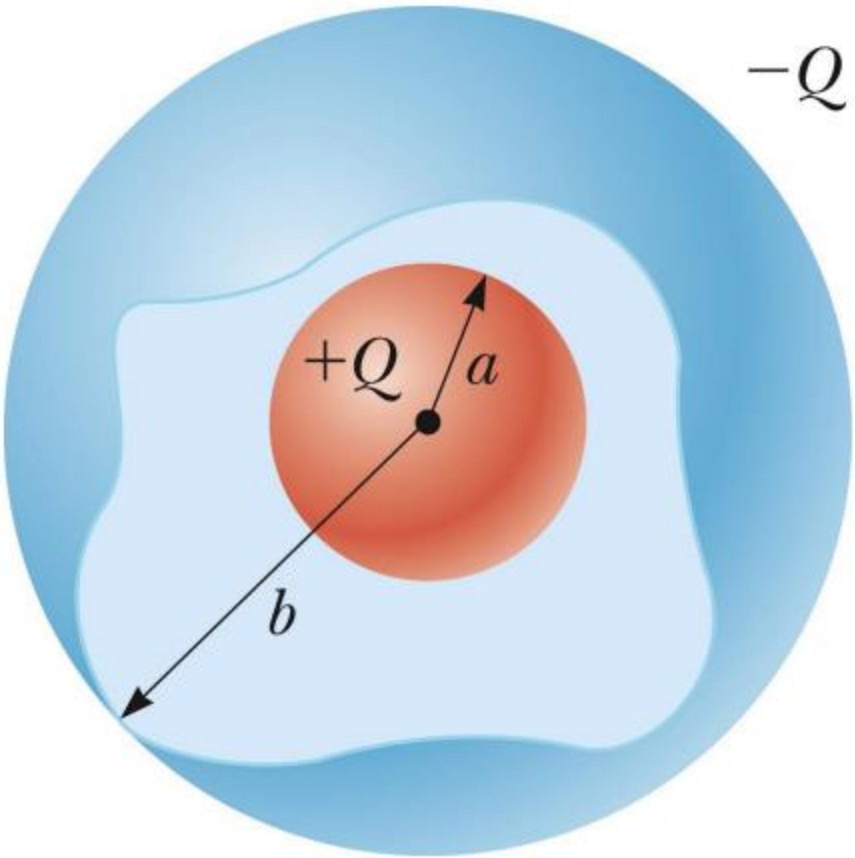
The Spherical Capacitor

- The potential difference will be

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

- The capacitance will be

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e\left(b - a\right)}$$



Calculating the Capacitance; A Spherical Capacitor:

$$\oint \vec{E}.\, d\vec{A} = \frac{q}{\varepsilon_0}$$

$$EA = E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2},$$

$$V = \int_{-}^{+} E\, ds = -\frac{q}{4\pi\varepsilon_0}\int_{b}^{a}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0}\frac{b-a}{ab},$$

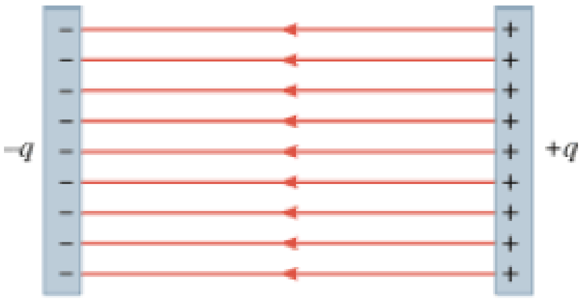
$$\Rightarrow \quad C = 4\pi\varepsilon_0 \frac{ab}{b-a} \qquad \text{(spherical capacitor).}$$

Capacitance – Isolated Sphere

- Assume a spherical charged conductor with radius *a.*
- The sphere will have the same capacitance as it would if there were a conducting sphere of infinite radius, concentric with the original sphere.
- The field lines that leave the surface of a positively charged isolated conductor must end somewhere; the walls of the room in which the conductor is housed can serve effectively as our sphere of infinite radius.

Energy Stored in an Electric Field

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.



(a)

Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other. The potential difference V' between the plates at that instant will be q'/C. If an extra increment of charge dq' is then transferred, the increment of work required will be,

$$dW = V' dq' = \frac{q'}{C} dq' .$$

The work required to bring the total capacitor charge up to a final value q is

$$W = \int dW = \frac{1}{C}\int_0^q q' dq' = \frac{q^2}{2C} .$$

This work is stored as potential energy U in the capacitor, so that

$$U = \frac{Q^2}{2C} = \tfrac{1}{2} Q \,\Delta V = \tfrac{1}{2} C (\Delta V)^2$$

Energy Density

The potential energy per unit volume between parallel-plate capacitor is

$(C = \varepsilon_0 A / d)$ $\Delta V = Ed$

$$U = \frac{1}{2}\frac{\epsilon_0 A}{d}(E^2 d^2) = \frac{1}{2}(\epsilon_0 A d)E^2$$

V = Ad (volume)

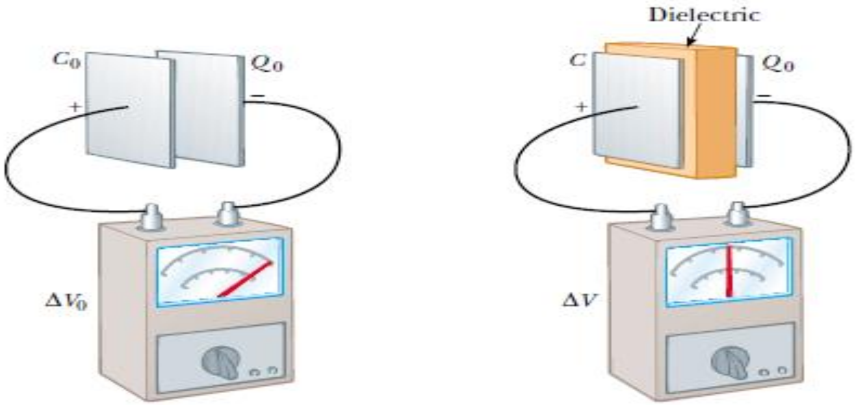
$$u = \frac{1}{2}\varepsilon_0 E^2 \text{ (energy density)} .$$

Example:

A 10 000μF capacitor is described as having a maximum working voltage of 25 V. Calculate the energy stored by the capacitor.

$$E = \frac{1}{2} CV^2 = \frac{1}{2} \times 10{,}000 \times 10^{-6} \times 25^2 = 3.125\ J$$

Capacitor with a Dielectric



$$\Delta V = \frac{\Delta V_0}{\kappa}$$

THE DIELECTRIC CONSTANT

The surface charges on the dielectric reduce the electric field inside the dielectric. This reduction in the electric field is described by the ***dielectric constant*** k, which is the ratio of the field magnitude E_0 without the dielectric to the field magnitude E inside the dielectric:

$$\kappa = \frac{E_0}{E}$$

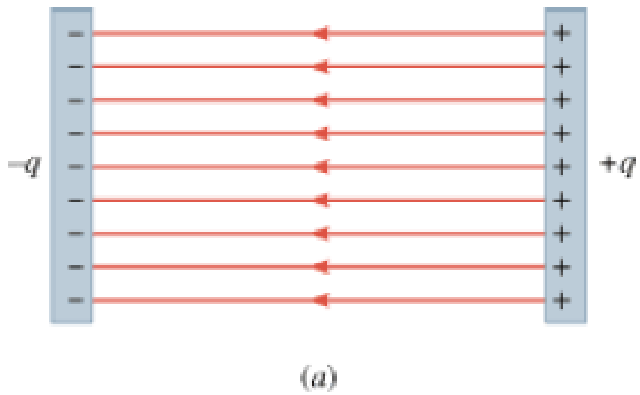
Every dielectric material has a characteristic *dielectric strength,* which is the maximum value of the electric field that it can tolerate without breakdown

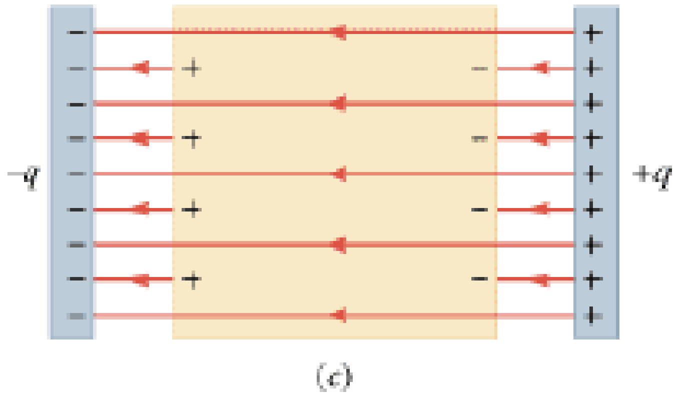
Some Properties of Dielectrics

Material	Dielectric Constant	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, $\kappa = \text{unity}$

Capacitance with a Dielectric





$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0}$$

$$\kappa = E_0 / E$$

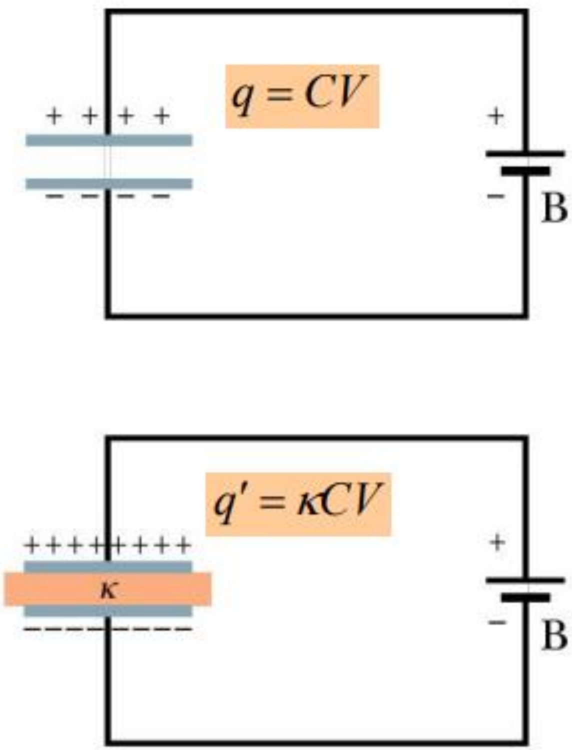
$$C = \kappa C_0$$

$$C = \kappa \frac{\epsilon_0 A}{d}$$

The capacitance with the dielectric present is increased by a factor of *k* over the capacitance without the dielectric.

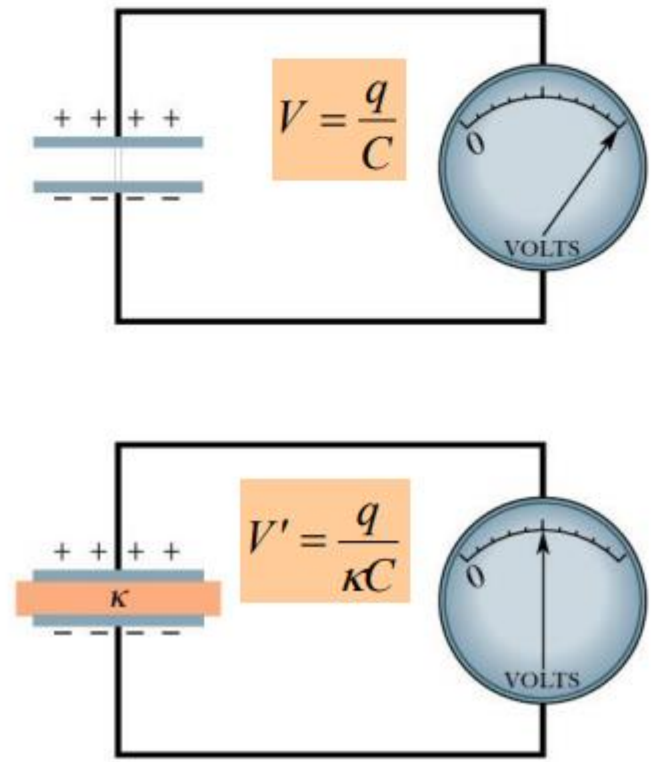
What Happens When You Insert a Dielectric?

- With battery attached, V=const, so more charge flows to the capacitor



V = a constant

- With battery disconnected, q=const, so voltage (for given q) drops.



q = a constant

Energy Stored Before the dielectric is inserted:

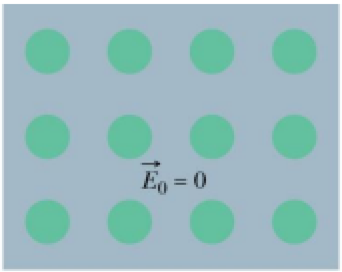
$$U_0 = \boxed{\frac{{Q_0}^2}{2C_0}}$$

Energy Stored After the dielectric is inserted:

$$U = \frac{{Q_0}^2}{2\kappa C_0} = \boxed{\frac{U_0}{\kappa}}$$

What Does the Dielectric Do?

- A dielectric material is made of molecules.
- Polar dielectrics already have a dipole moment (like the water molecule).
- Non-polar dielectrics are not naturally polar, but actually stretch in an electric field, to become polar.
- The molecules of the dielectric align with the applied electric field in a manner to oppose the electric field.
- This reduces the electric field, so that the net electric field is less than it was for a given charge on the plates.
- This lowers the potential (case b of the previous slide).
- If the plates are attached to a battery (case a of the previous slide), more charge has to flow onto the plates.



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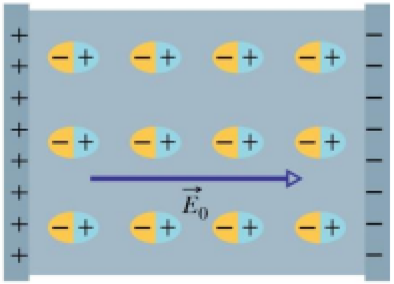

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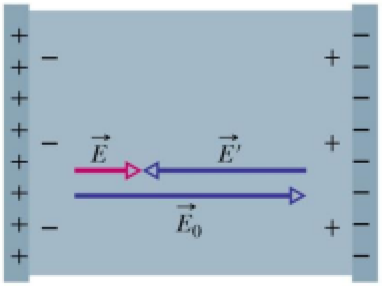


Example 26.6 **A Paper-Filled Capacitor**

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper.

Find its capacitance.

Solution Because $\kappa = 3.7$ for paper (see Table 26.1), we have

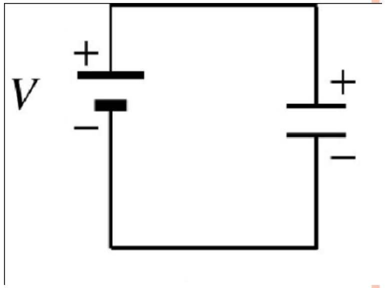
$$C = \kappa \frac{\epsilon_0 A}{d}$$

$$= 3.7\left(\frac{(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2})(6.0 \times 10^{-4}\,\mathrm{m^2})}{1.0 \times 10^{-3}\,\mathrm{m}}\right)$$

$$= 20 \times 10^{-12}\,\mathrm{F} = \boxed{20\,\mathrm{pF}}$$

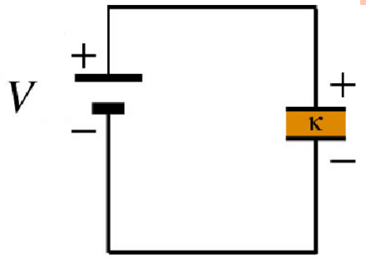
Capacitor with Dielectric

- *Question 1:*
- Consider a parallel plate capacitor with capacitance C = 2.00 μF connected to a battery with voltage V = 12.0 V as shown. What is the charge stored in the capacitor?



$$q = CV = \left(2.00 \cdot 10^{-6}\ \mathrm{F}\right)\left(12.0\ \mathrm{V}\right) = 2.40 \cdot 10^{-5}\ \mathrm{C}$$

- *Question 2:*
- Now insert a dielectric with dielectric constant κ = 2.5 between the plates of the capacitor. What is the charge on the capacitor?



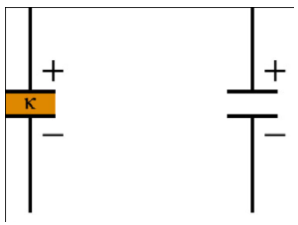
$C = \kappa C_{air}$ The capacitance of the capacitor is increased

$$q = CV = 2.50 \times 2.0 \cdot 10^{-6}\,\mathrm{F} \times 12.0\,\mathrm{V} = 6.0 \cdot 10^{-5}\,\mathrm{C}$$

The additional charge is provided by the battery.

Capacitor with Dielectric (2)

- *Question 3:*
- We isolate the charged capacitor with a dielectric by disconnecting it from the battery. We remove the dielectric, keeping the capacitor isolated.
- What happens to the charge and voltage on the capacitor?



- The charge on the isolated capacitor cannot change because there is nowhere for the charge to flow. ***Q remains constant***.
- The voltage on the capacitor will be

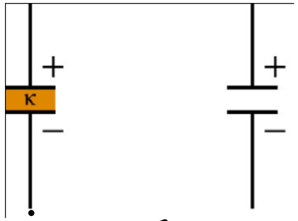
$$V = \frac{q}{C} = \frac{6.00 \cdot 10^{-5}\ \text{C}}{2.00 \cdot 10^{-6}\ \text{F}} = 30.0\ \text{V}$$

V increases

- The voltage went up because removing the dielectric increased the electric field and the resulting potential difference between the plates.

Capacitor with Dielectric (3)

- *Question 4:*
- Does removing the dielectric from the isolated capacitor change the energy stored in the capacitor?



- The energy stored in the capacitor before the dielectric was removed was

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_{air}V^2 = \frac{1}{2}(2.50)\left(2.00\cdot 10^{-6}\text{ F}\right)(12\text{ V})^2 = 3.60\cdot 10^{-4}\text{ J}$$

- After the dielectric is removed, the energy is

$$U = \frac{1}{2}C_{air}V^2 = \frac{1}{2}\left(2.00\cdot 10^{-6}\text{ F}\right)(30\text{ V})^2 = 9.00\cdot 10^{-4}\text{ J}$$

- The energy increases --- we must add energy to pull out the dielectric. [Or, the polarized dielectric is sucked into the **E**.]

Example

- Given a 7.4 pF air-filled capacitor. You are asked to convert it to a capacitor that can store up to 7.4 μJ with a maximum voltage of 652 V. What dielectric constant should the material have that you insert to achieve these requirements?

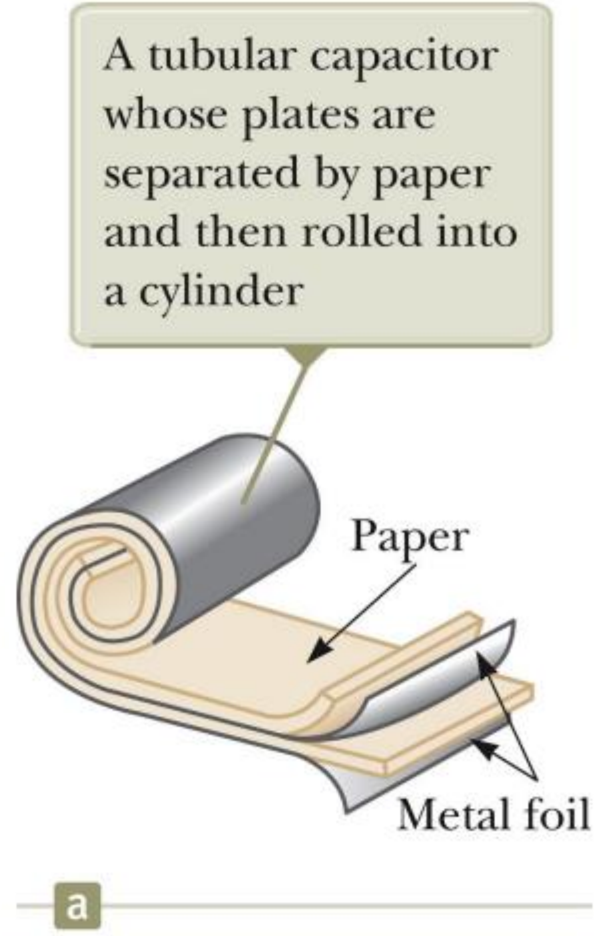
- Key Idea: The capacitance with the dielectric in place is given by $C=\kappa C_{air}$

and the energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_{air}V^2$

So, $$\kappa = \frac{2U}{C_{air}V^2} = \frac{2(7.4 \cdot 10^{-6} J)}{(7.4 \cdot 10^{-12} F)(652V)^2} = 4.7$$

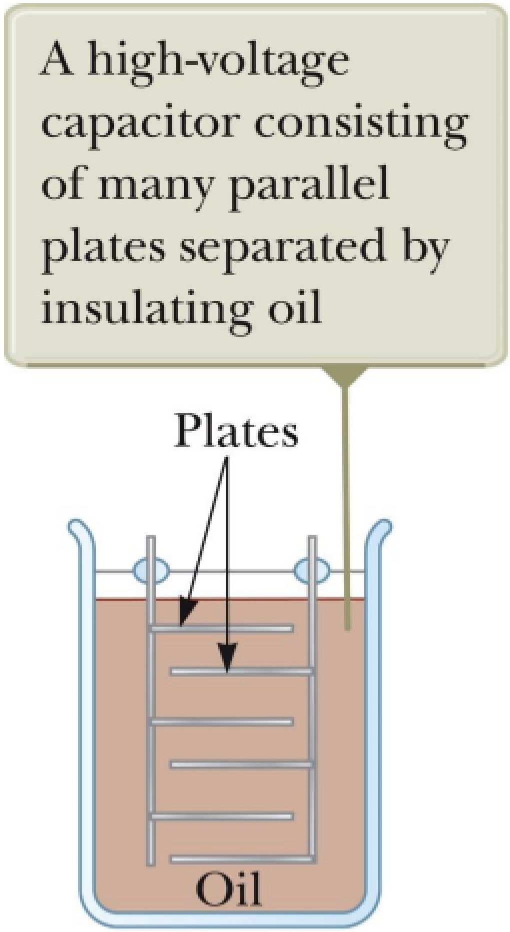
Types of Capacitors – Tubular

- Metallic foil may be interlaced with thin sheets of paraffin-impregnated paper or Mylar.
- The layers are rolled into a cylinder to form a small package for the capacitor.



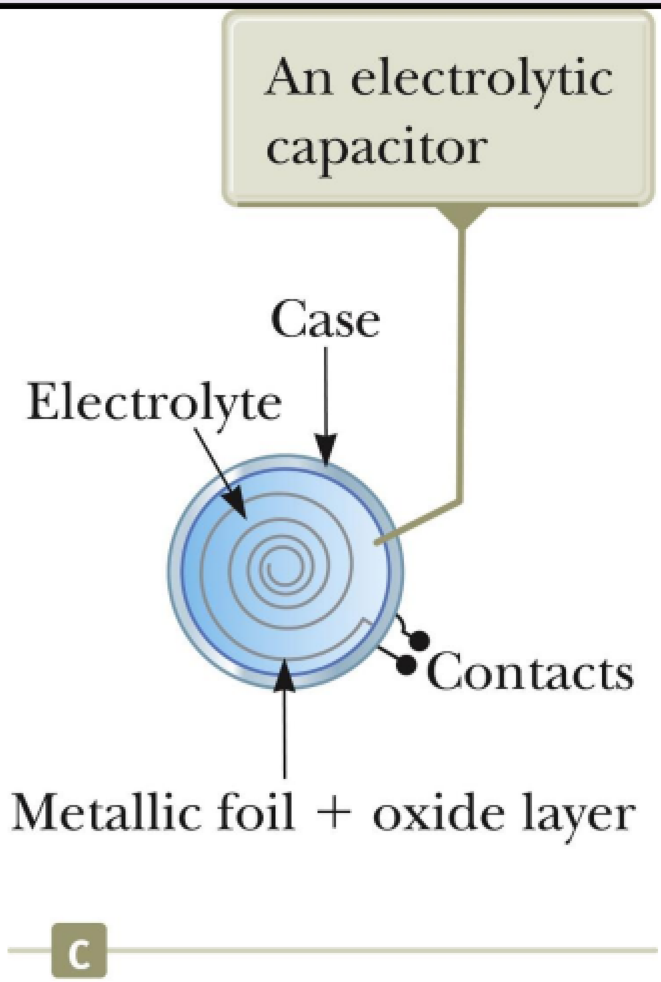
Types of Capacitors – Oil Filled

- Common for high-voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil.



Types of Capacitors – Electrolytic

- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution.
- When a voltage is applied between the foil and the electrolyte, a thin layer of metal oxide is formed on the foil.
- This layer serves as a dielectric.
- Large values of capacitance can be obtained because the dielectric layer is very thin and the plate separation is very small.



Types of Capacitors – Variable

- •Variable capacitors consist of two interwoven sets of metallic plates.
- •One plate is fixed and the other is movable.
- •Contain air as the dielectric
- •These capacitors generally vary between 10 and 500 pF.
- •Used in radio tuning circuits

When one set of metal plates is rotated so as to lie between a fixed set of plates, the capacitance of the device changes.

CLICKER QUESTION - PART 1

- A parallel-plate air-filled capacitor has a capacitance of 50 pF.
- (a) If each of the plates has an area of $A=0.35\ m^2$, what is the separation?

$\varepsilon_0=8.85\ 10^{-12}\ C^2/Nm^2$

A) $12.5\ 10^{-1}$ m

B) $6.2\ 10^{-2}$ m

C) 1.3 m

- Find the capacitance of a 4.0 cm diameter sensor immersed in oil if the plates are separated by 0.25 mm.

$$C = 8.85\times10^{-12}\,\text{F/m}\left(\frac{\varepsilon_r A}{d}\right) \qquad (\varepsilon_r = 4.0 \text{ for oil})$$

The plate area is $A = \pi r^2 = \pi\left(0.02\ \text{m}^2\right) = 1.26\times10^{-3}\ \text{m}^2$

$$C = 8.85\times10^{-12}\,\text{F/m}\left(\frac{(4.0)\left(1.26\times10^{-3}\ \text{m}^2\right)}{0.25\times10^{-3}\ \text{m}}\right) = 178\ \text{pF}$$

- Example: Given a 1 farad parallel plate capacitor having a plate separation of 1mm. What is the area of the plates?

$$C = \varepsilon_0 \frac{A}{d}$$

$$A = \frac{C\,d}{\varepsilon_0} = \frac{(1.0F)(1.0 \times 10^{-3} m)}{8.85 \times 10^{-12} F/m}$$

$$= 1.1 \times 10^{8} m^{2}$$

Clicker Question - part 1

- A parallel-plate air-filled capacitor has a capacitance of 50 pF.
- (a) If each of the plates has an area of A=0.35 m^2, what is the separation?

B) 6.2 10^{-2} m

Use $C = \frac{\epsilon_0 A}{d}$ to solve for d:

$$d = \frac{\epsilon_0 A}{C} = \frac{(8.85 \cdot 10^{-12} \frac{C^2}{Nm^2})(0.35 m^2)}{50 \cdot 10^{-12} F} = 6.2 \cdot 10^{-2} m$$

Clicker Question - part 2

- An air-filled parallel plate capacitor has a capacitance of 50pF.
- (b) If the region between the plates is now filled with material having a dielectric constant of $\kappa=2$, what is the capacitance?

A) the same

B) 25 pF

C) 100 pF

Lect ure

Clicker Question - part 2

- A air-filled parallel plate capacitor has a capacitance of 50 pF.
- (b) If the region between the plates is now filled with material having a dielectric constant of κ=2, what is the capacitance?

C) 100 pF

$$C_{new} = C \cdot \kappa = 50pF \cdot 2 = 100pF$$

Example, Capacitors in Parallel and in Series:

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu\text{F}, \quad C_2 = 5.30\ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50\ \mu\text{F}.$$

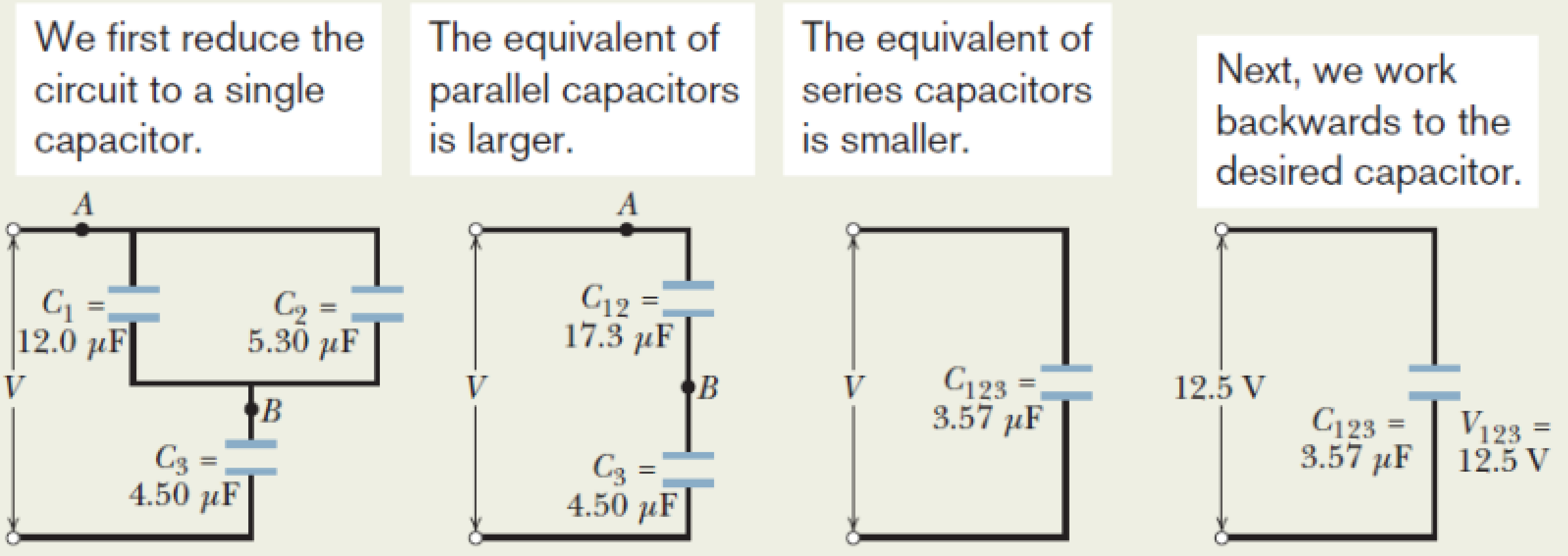
⇨

$$C_{12} = C_1 + C_2 = 12.0\ \mu\text{F} + 5.30\ \mu\text{F} = 17.3\ \mu\text{F}.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$= \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}} = 0.280\ \mu\text{F}^{-1},$$

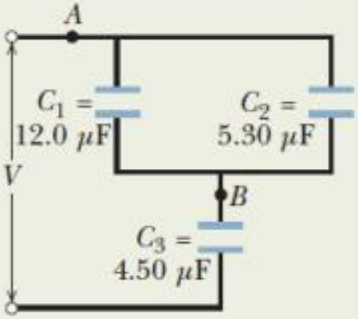
$$C_{123} = \frac{1}{0.280\ \mu\text{F}^{-1}} = 3.57\ \mu\text{F}. \qquad \text{(Answer)}$$



(a) (b) (c) (d)

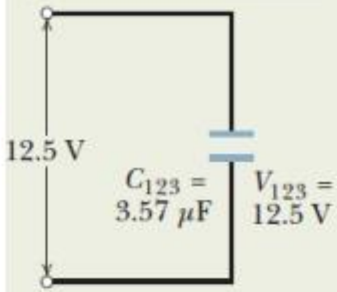
(b) The potential difference applied to the input terminals in Fig. 25-10*a* is $V = 12.5$ V. What is the charge on C_1?

We first reduce the circuit to a single capacitor.



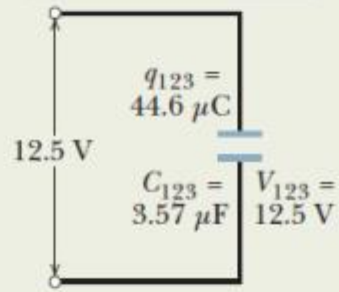
(*a*)

Next, we work backwards to the desired capacitor.



(*d*)

Applying $q = CV$ yields the charge.



(*e*)

$$q_{123} = C_{123}V = (3.57\ \mu\text{F})(12.5\ \text{V}) = 44.6\ \mu\text{C}.$$

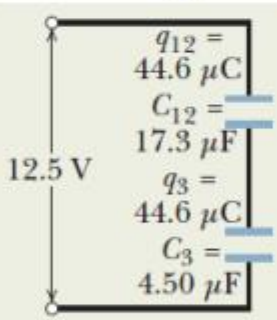
$$q_{12} = q_{123} = 44.6\ \mu\text{C}.$$

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\ \mu\text{C}}{17.3\ \mu\text{F}} = 2.58\ \text{V}.$$

$$V_1 = V_{12} = 2.58\ \text{V},$$

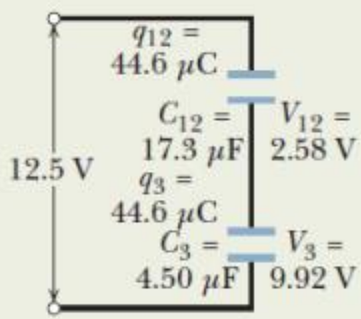
$$q_1 = C_1V_1 = (12.0\ \mu\text{F})(2.58\ \text{V}) = 31.0\ \mu\text{C}.$$

Series capacitors and their equivalent have the same q ("seri-q").



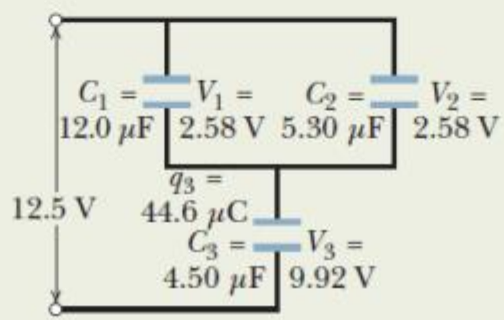
(*f*)

Applying $V = q/C$ yields the potential difference.



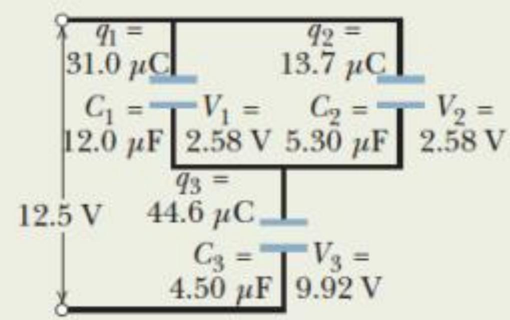
(*g*)

Parallel capacitors and their equivalent have the same V ("par-V").



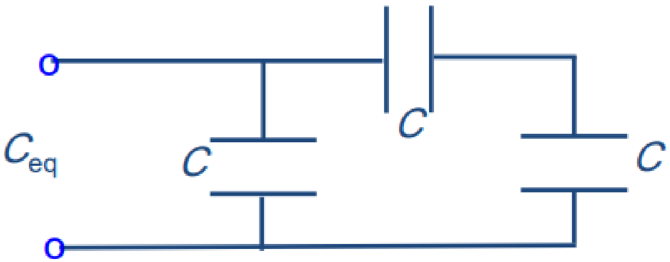
(*h*)

Applying $q = CV$ yields the charge.



(*i*)

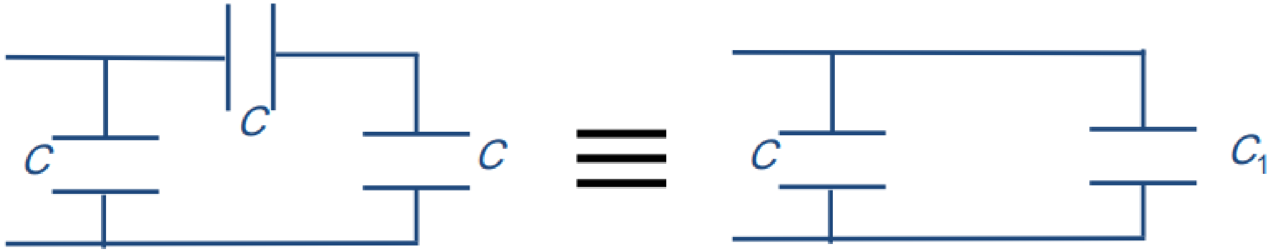
What is the equivalent capacitance, C_{eq}, of the combination shown?



(a) $C_{eq} = (3/2)C$

(b) $C_{eq} = (2/3)C$

(c) $C_{eq} = 3C$



$$\frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_1 = \frac{C}{2} \Rightarrow C_{eq} = C + \frac{C}{2} = \frac{3}{2}C$$

Potential Energy and Energy Density of an Electric Field:

An isolated conducting sphere whose radius R is 6.85 cm has a charge $q = 1.25$ nC.

(a) How much potential energy is stored in the electric field of this charged conductor?

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\varepsilon_0 R}$$

$$= \frac{(1.25 \times 10^{-9}\ \mathrm{C})^2}{(8\pi)(8.85 \times 10^{-12}\ \mathrm{F/m})(0.0685\ \mathrm{m})}$$

$$= 1.03 \times 10^{-7}\ \mathrm{J} = 103\ \mathrm{nJ}. \qquad \text{(Answer)}$$

(b) What is the energy density at the surface of the sphere?

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}.$$

The energy density is then

$$u = \tfrac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4}$$

$$= \frac{(1.25 \times 10^{-9}\ \text{C})^2}{(32\pi^2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0685\ \text{m})^4}$$

$$= 2.54 \times 10^{-5}\ \text{J/m}^3 = 25.4\ \mu\text{J/m}^3. \qquad \text{(Answer)}$$

Suppose the capacitor shown here is charged to Q and then the battery is *disconnected*

Now suppose you pull the plates further apart so that the final separation is d_1

Which of the quantities Q, C, V, U, E change?

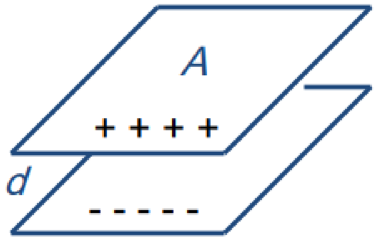
Q: Charge on the capacitor does not change

C: Capacitance Decreases

V: Voltage Increases

U: Potential Energy Increases

E: Electric Field does not change



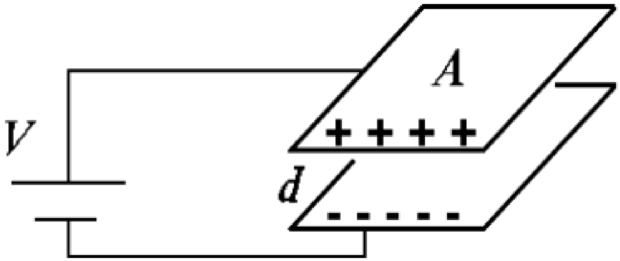
How do these quantities change?

Answers: $C_1 = \frac{d}{d_1} C$ $V_1 = \frac{d_1}{d} V$ $U_1 = \frac{d_1}{d} U$

Suppose the battery (V) is kept attached to the capacitor

Again pull the plates apart from d to d_1

Now which quantities, if any, change?



Q: Charge Decreases

C: Capacitance Decreases

V: Voltage on capacitor does not change

U: Potential Energy Decreases

E: Electric Field Decreases

How much do these quantities change?

Answers: $Q_1 = \frac{d}{d_1} Q$ $C_1 = \frac{d}{d_1} C$ $U_1 = \frac{d}{d_1} U$ $E_1 = \frac{d}{d_1} E$

Circuit Symbols

- A circuit diagram is a simplified representation of an actual circuit.
- Circuit symbols are used to represent the various elements.
- Lines are used to represent wires.
- The battery's positive terminal is indicated by the longer line.