Current and Resistance

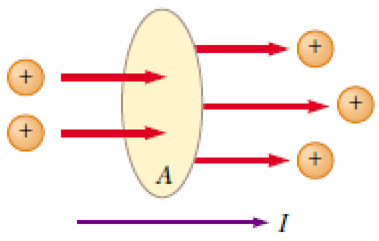
CHAPTER OUTLINE

27.1 Electric Current

The current is the rate at which charge flows through this surface

If ΔQ *is the amount of charge that passes through this area in a time interval* Δt*, the average current* I_{av} *is equal to the charge that passes through A per unit time:*

$$I_{av} = \frac{\Delta Q}{\Delta t}$$



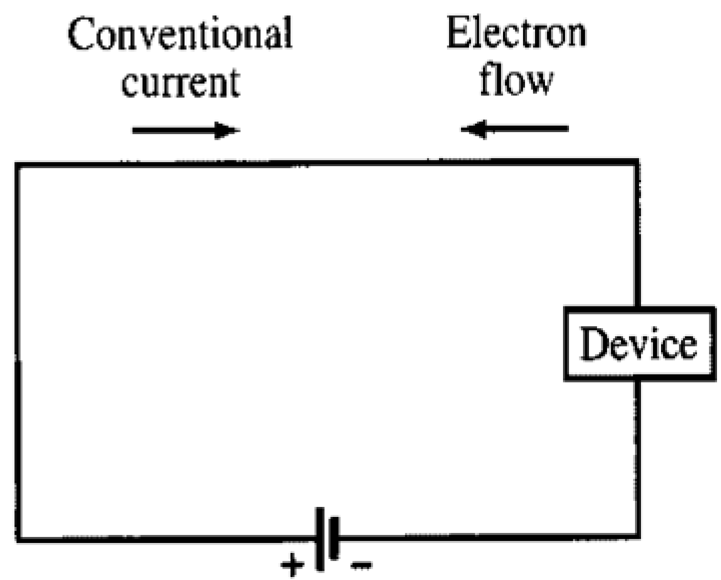
If the rate at which charge flows varies in time, then the current varies in time; we define the **instantaneous current** I as the differential limit of average current:

$$I \equiv \frac{dQ}{dt}$$

The SI unit of current is the **ampere** (A):

$$1\text{ A} = \frac{1\text{ C}}{1\text{ s}}$$

The direction of the current is opposite the direction of flow of electrons

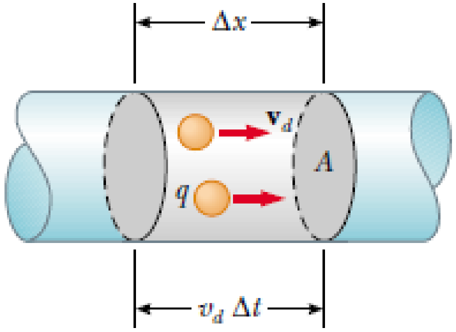


Microscopic Model of Current

Consider the current in a conductor of cross-sectional area *A.The volume of a section of the conductor of length* Δx is *A* Δ *x. If n represents the number of mobile charge* carriers per unit volume (in other words, the charge carrier density), the number of carriers in the gray section is *nA* Δ *x.Therefore, the total charge* Δ *Q in this section is:*

$$\Delta Q = (nAv_d\,\Delta t)\,q$$

$$I_{av} = \frac{\Delta Q}{\Delta t} = nqv_d A$$



27.2 Resistance

The current density *J in the conductor is defined as the current per unit area*

$$I = nqv_d A$$

$$J \equiv \frac{I}{A} = nqv_d$$

A current density J and an electric field E are established in a conductor whenever a potential difference is maintained across the conductor

$$\mathbf{J} = \sigma \mathbf{E}$$

$$\Delta V = E\ell$$

$$J = \sigma E = \sigma \frac{\Delta V}{\ell}$$

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = RI$$

The quantity $R = \ell/\sigma A$ is called the **resistance** of the conductor.

$$R \equiv \frac{\Delta V}{I}$$

SI units of the resistance:

$$1\ \Omega \equiv \frac{1\ \text{V}}{1\ \text{A}}$$

The inverse of conductivity is **resistivity**[3] ρ:

$$\rho = \frac{1}{\sigma}$$

where ρ has the units ohm-meters $(\Omega \cdot \mathrm{m})$. Because $R = \ell/\sigma A$, we can express the resistance of a uniform block of material along the length ℓ as

$$R = \rho \frac{\ell}{A}$$

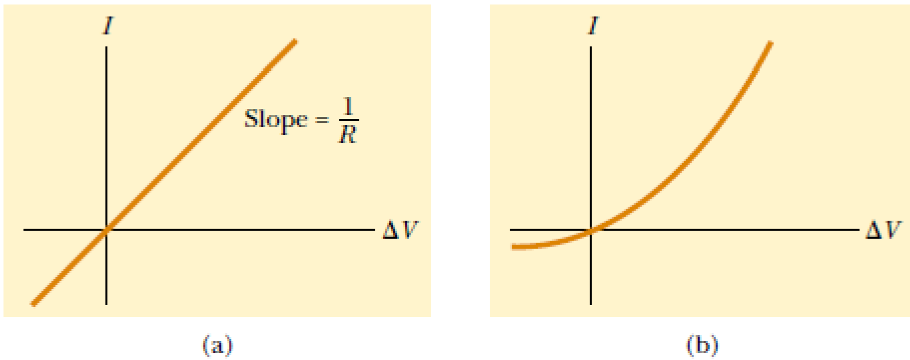


Figure 27.7 (a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor. (b) A nonlinear current–potential difference curve for a junction diode. This device does not obey Ohm's law.

1) Calculate the resistance of a uniform wire of diameter 0.32 mm and length 5.0m
The resistivity of the wire $= 5.0 \times 10^{-7}\ \Omega m$

$R = 5.0 \times 10^{-7} \times 5/(\text{pi} \times 0.0016^2)$

$5/(\text{pi} \times 0.00000256)$

$5/8\times10^{-6}$

625000

$R = 31\ \Omega$

2) Calculate the resistance of a rectangle strip of copper length 0.08m, thickness 15 mm and width 0.80mm The resistivity of copper $= 1.7 \times 10^{-8}\ \Omega m$

Area = 0.015×0.0080

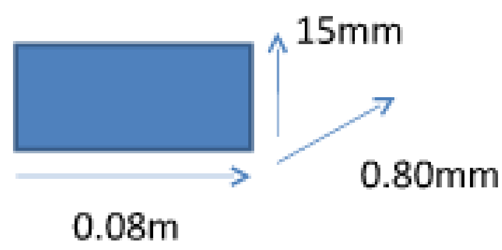
$A = 0.00012$

$L = 0.08m$

$R = 1.7 \times 10^{-8} \times 0.08 / 0.00012$

$R = 0.0000113$

$R = 11\ \mu\Omega$



3) A wire of uniform diameter 0.28 mm and length 1.50m has a resistance of 45 Ω

Calculate

a)Its resistivity

b) The length of wire that has the resistance of 1.0Ω

$$\rho = R\frac{A}{\ell}$$

r = 0.14 mm

r= 0.00014m

L = 1.50m

R = 45 Ω

A = 6.15×10^{-8}

ρ =45 * (6.15×10^{-8} / 1.5)

= 0.0000018

=1.8×10^{-6} Ωm

A = pi r^2

27.4 Resistance and Temperature

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

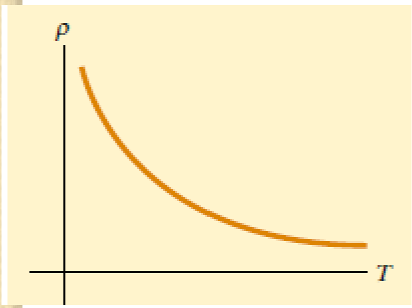
where ρ is the resistivity at some temperature T (in degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the **temperature coefficient of resistivity.** From Equation 27.19, we see that the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

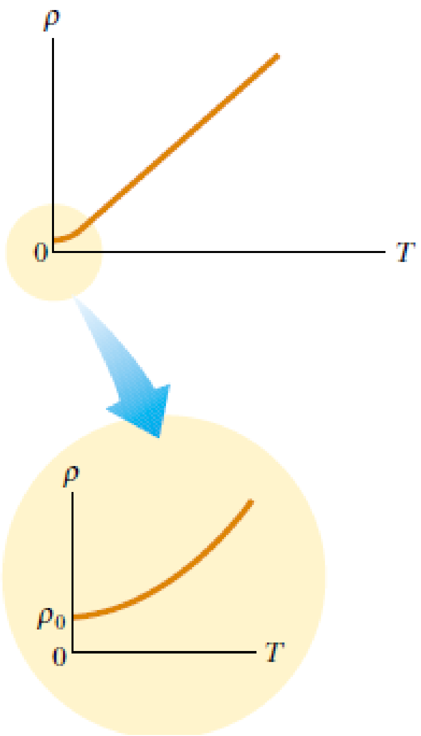
Note that the unit for α is degrees Celsius^{-1}

we can write the variation of resistance as

$$R = R_0[1 + \alpha(T - T_0)]$$



Resistivity versus temperature for a pure semiconductor, such as silicon or germanium.



Resistivity versus temperature for a metal such as copper. The curve is linear over a wide range of temperatures, and ρ increases with increasing temperature. As *T approaches* absolute zero (inset), the resistivity approaches a finite value ρ0.

Example 27.6 **A Platinum Resistance Thermometer**

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

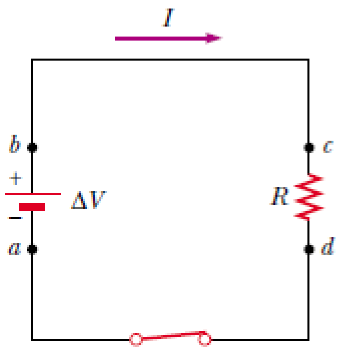
$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3} (°\text{C})^{-1}](50.0\ \Omega)}$$

$$= 137°\text{C}$$

Because $T_0 = 20.0°\text{C}$, we find that T, the temperature of the melting indium sample, is 157°C.

27.6 Electrical Power

$$\frac{dU}{dt} = \frac{d}{dt}(Q\,\Delta V) = \frac{dQ}{dt}\,\Delta V = I\,\Delta V$$



$$\mathcal{P} = I\,\Delta V$$

$$\mathcal{P} = I^2 R = \frac{(\Delta V)^2}{R}$$

When I is expressed in amperes, ΔV in volts, and R in ohms, the SI unit of power is the **watt**

Quick Quiz 27.7 The same potential difference is applied to the two lightbulbs shown in Figure 27.14. Which one of the following statements is true? (a) The 30-W bulb carries the greater current and has the higher resistance. (b) The 30-W bulb carries the greater current, but the 60-W bulb has the higher resistance. (c) The 30-W bulb has the higher resistance, but the 60-W bulb carries the greater current. (d) The 60-W bulb carries the greater current and has the higher resistance.

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Figure 27.14 (Quick Quiz 27.7) These lightbulbs operate at their rated power only when they are connected to a 120-V source.

Quick Quiz 27.8 For the two lightbulbs shown in Figure 27.15, rank the current values at points *a* through *f*, from greatest to least.

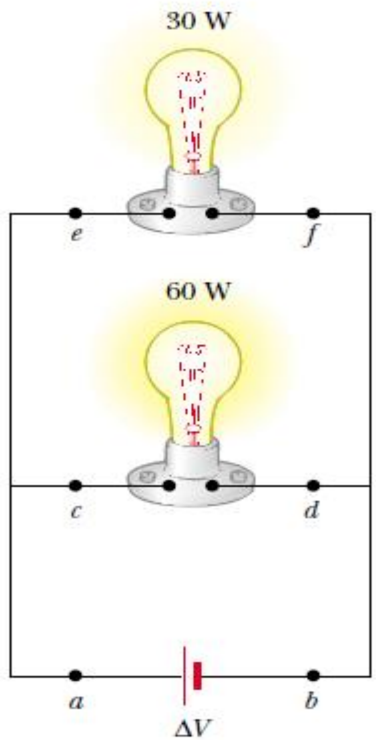

Figure 27.15 (Quick Quiz 27.8) Two lightbulbs connected across the same potential difference.

Example 27.7 **Power in an Electric Heater**

An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. Find the current carried by the wire and the power rating of the heater.

Because $\Delta V = IR$, we have

$$I = \frac{\Delta V}{R} = \frac{120\ \text{V}}{8.00\ \Omega} = \boxed{15.0\ \text{A}}$$

We can find the power rating using the expression $\mathcal{P} = I^2R$:

$$\mathcal{P} = I^2R = (15.0\ \text{A})^2(8.00\ \Omega) = 1.80 \times 10^3\ \text{W}$$

$$\mathcal{P} = \boxed{1.80\ \text{kW}}$$