Phys 103
Chapter 3

Vectors

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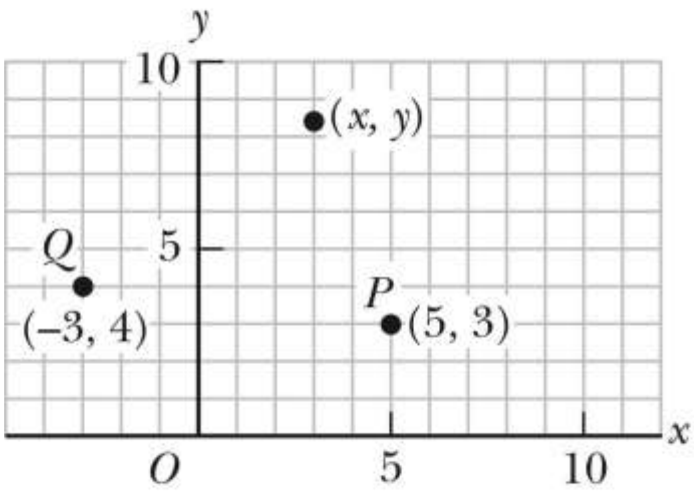
LECTURE OUTLINE

- 3.1 Coordinate Systems
- 3.2 Vector and Scalar Quantities
- 3.3 Some Properties of Vectors
- 3.4 Components of a Vector and Unit Vectors

Cartesian Coordinate System

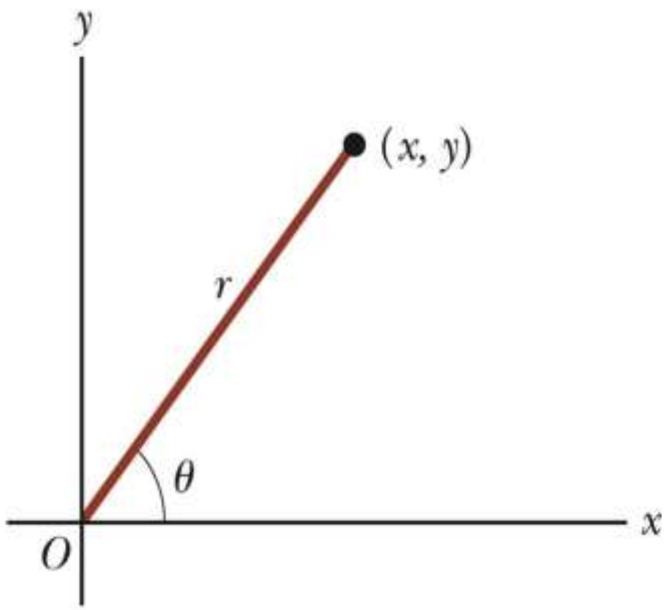
- Used to describe the position of a point in a space
- Coordinate system consists of
  - a fixed reference point called the origin
  - specific axes with scales and labels



- Instructions on how to label a point relative to the origin and the axes
- Also called rectangular coordinate system
- x - and y - axes intersect at the origin
- Labeled are Points (x,y)

Polar Coordinate System (r,θ)

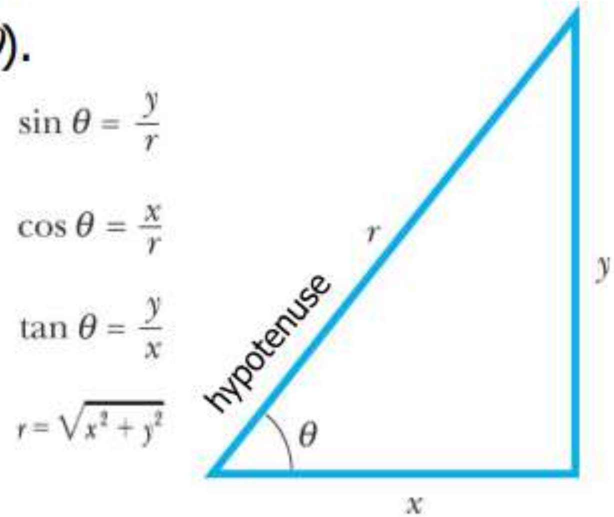
- represent a point in a plan
- Origin and reference line are noted
- Point is distance r from the origin in the direction of angle θ, counterclockwise from positive x-axis
- Points are labeled as (r,θ)



Polar to Cartesian Coordinates $(r, \theta) \longrightarrow (x, y)$

The right triangle used to relate (x, y) to (r, θ).

- Based on forming a right triangle from r and θ
- $x = r \cos \theta$
- $y = r \sin \theta$



4- What are the Cartesian coordinates of the point (r, θ) = (3, 60 º)?

؟(r, θ) = (3, 60 º)

a.(1.8, 2.1) b. (1.5, 2.6) c. (1.5, 3.6) d. (1.5, 3)

$x = r \cos \theta = 3 \times ½ = 1.5$

$y = r \sin \theta = = 3 \times 0.866 = 2.6$

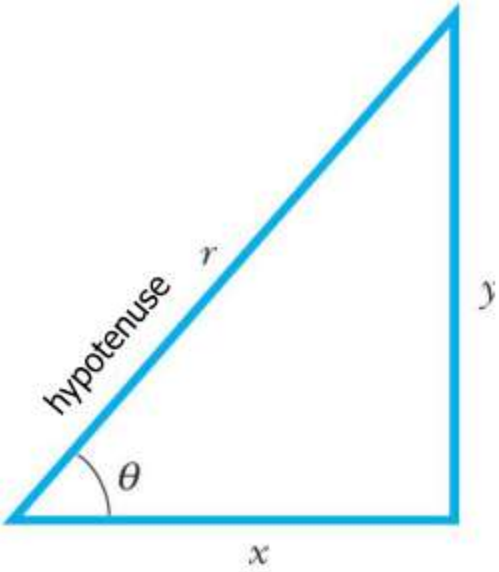
Cartesian to Polar Coordinates $(x, y) \longrightarrow (r, \theta)$

- r is the hypotenuse and θ an angle

$$\tan\theta = \frac{y}{x}$$

$$r = \sqrt{x^2 + y^2}$$

- θ must be counterclockwise from positive x axis for these equations to be valid



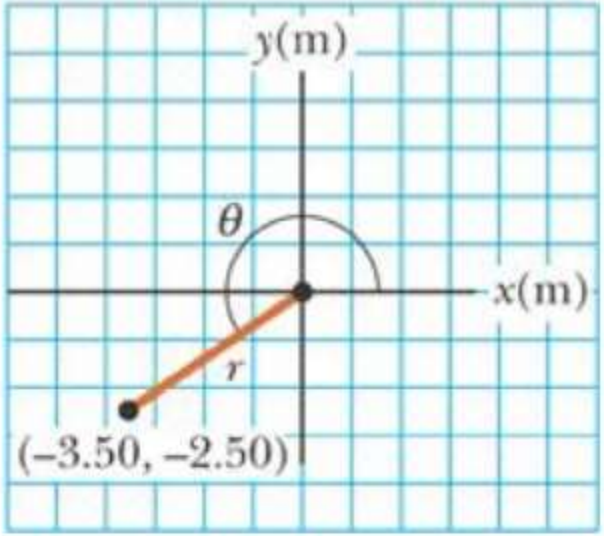
Example:

3- The polar coordinates (r, θ) of the Cartesian coordinates (-3.5,-2.5) is:

a. (4.3, 216°) b. (10, 100°) c. (120, 17°) d. (30, 150°)

Example 3.1

- The Cartesian coordinates of a point in the *xy* plane are $(x,y) = (-3.50, -2.50)$ m, as shown in the figure. Find the polar coordinates of this point. (r,θ)



- **Solution:**

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\ \text{m})^2 + (-2.50\ \text{m})^2} = 4.30\ \text{m}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.50\ \text{m}}{-3.50\ \text{m}} = 0.714$$

$\tan^{-1} 0.714 = 36°$

$\theta = 36+180 = 216°$

Vectors and Scalars

- A ***scalar quantity***

is completely specified by a single value with an appropriate unit and has no direction.

- A ***vector quantity***

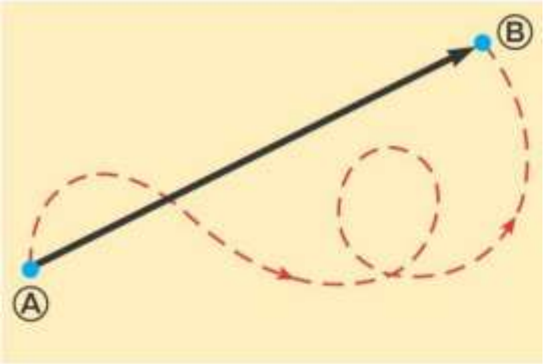
is completely described by a number and appropriate units plus a direction.

Vector Notation

- When handwritten, use an arrow: $\vec{A}$
- When printed, will be in bold print: **A**
- The magnitude of the vector has physical units A or $|\vec{A}|$
- The magnitude of a vector is always a positive number

Vector Example

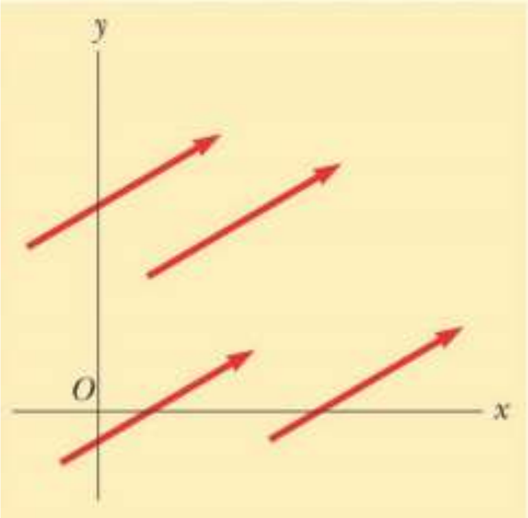
- A particle travels from A to B along the path shown by the dotted red line
- This is the distance traveled and is a scalar



- The displacement is the solid line from A to B
- The displacement is independent of the path taken between the two points
- Displacement is a vector

Equality of Two Vectors

- Two vectors are ***equal*** if they have the same magnitude and the same direction
- **A** = **B** if *A* = *B* and they point along parallel lines
- All of the vectors shown are equal



Adding Vectors

- When adding vectors, their directions must be taken into account
- Units must be the same

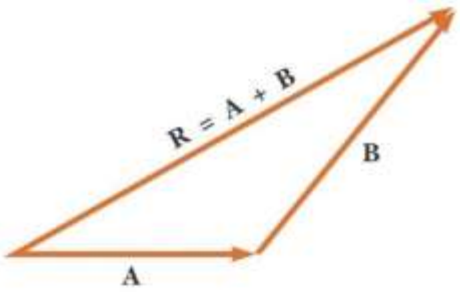
There are two ways to adding Vectors

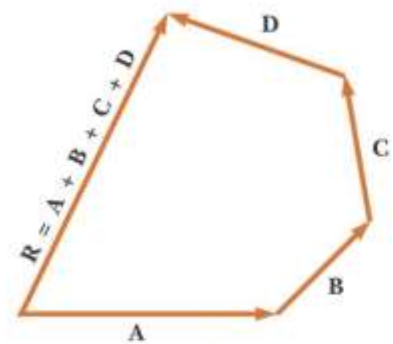
(1) Graphical Methods: Use scale drawing

(2) Algebraic Methods: More convenient

Adding Vectors Graphically

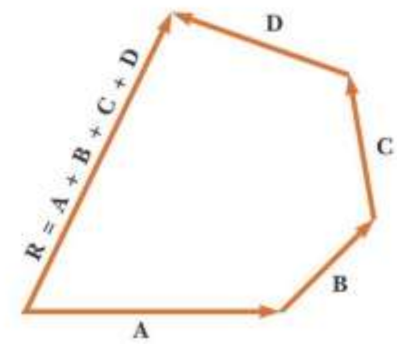
- Choose a scale
- Draw the first vector with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector B with the appropriate length and in the direction specified, with respect to a coordinate system. Its origin is the end of vector A
- Continue drawing the vectors "tip-to-tail"
- The resultant is drawn from the origin of **A** to the end of the last vector
- Measure the length of **R** and its angle





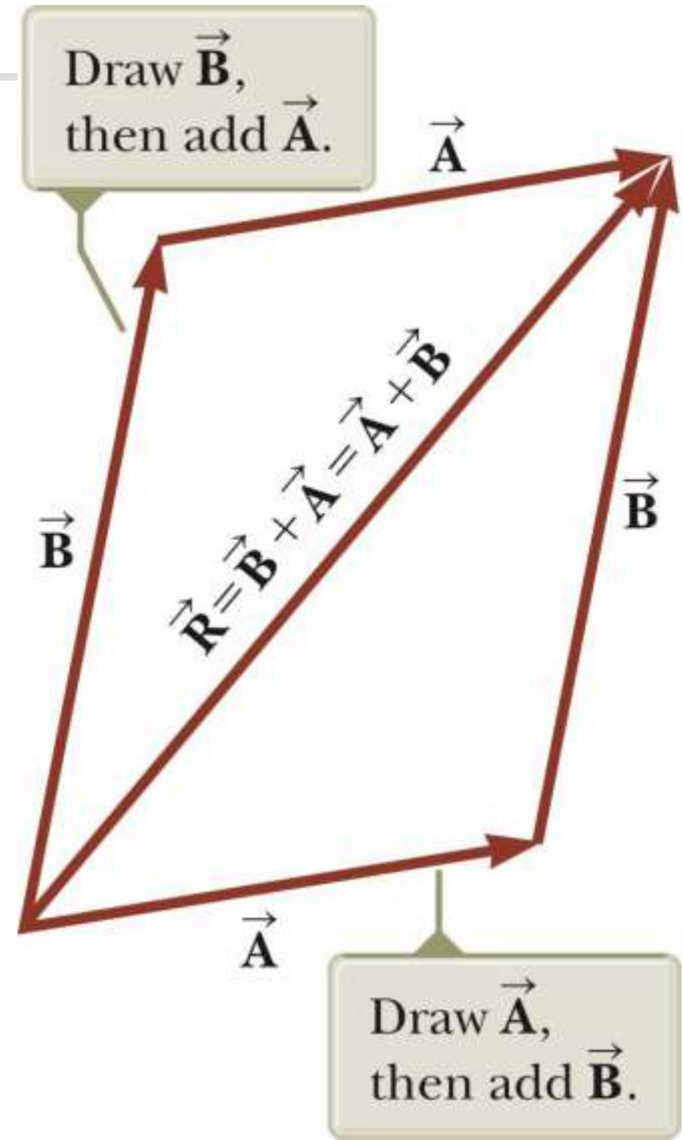
Adding Vectors Graphically

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



Adding Vectors, Rules

- When two vectors are added, the sum is independent of the order of the addition .
  - This is the commutative law of addition
    - A + B = B + A

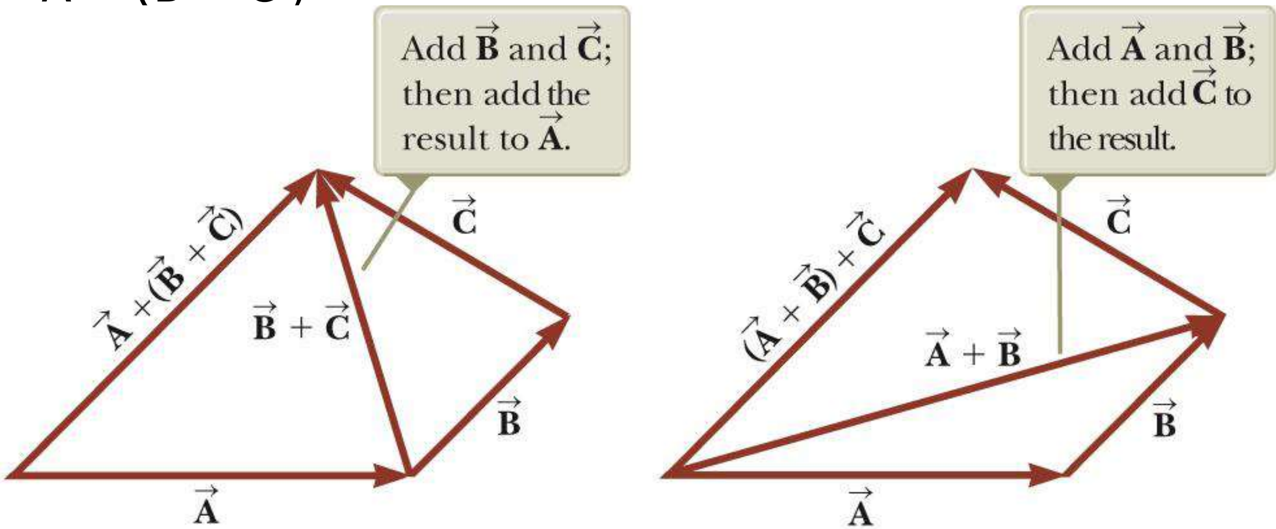


.Adding Vectors, Rules cont

- When adding three or more vectors, their sum is independent of the way in which the individual vectors are grouped

- This is called **the Associative Property of Addition**

  - (A + B) + C = A + (B + C)



Adding Vectors

- When adding vectors, all of the vectors must have
  - ✓ the same units
  - ✓ the same type of quantity (cannot add a displacement to a velocity)

- **The negative of a vector:**

  is defined as the vector that, when added to the original vector, gives a resultant of zero

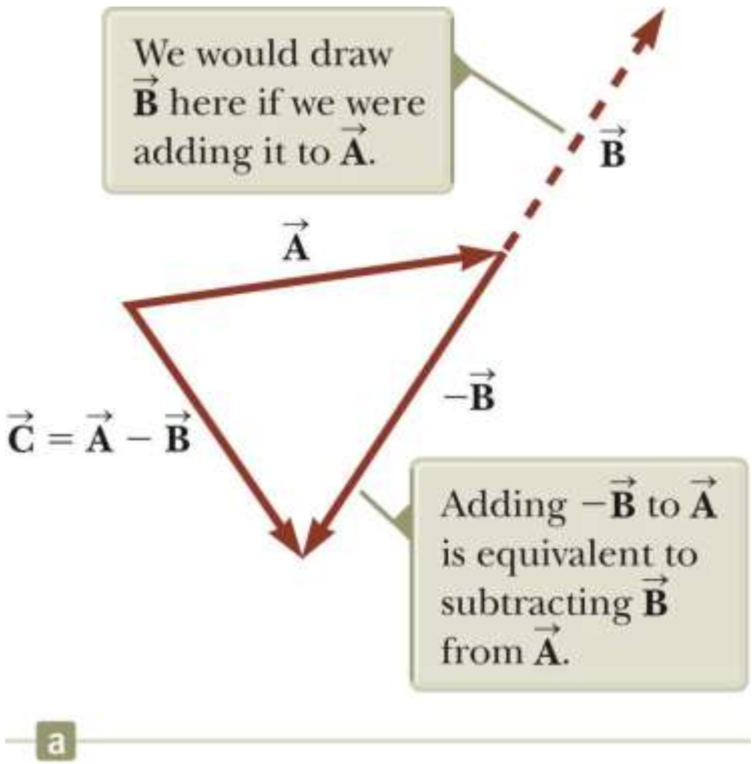
  - Represented as –**A**
  - **A** + (-**A**) = 0

- The negative of the vector will have the same magnitude, but point in the opposite direction

Subtracting Vectors

If **A** – **B**, then use **A**+(**-B**)

A – **B** = **A**+(**-B**)



Multiplying or Dividing a Vector by a Scalar

- The result of the multiplication or division is a vector

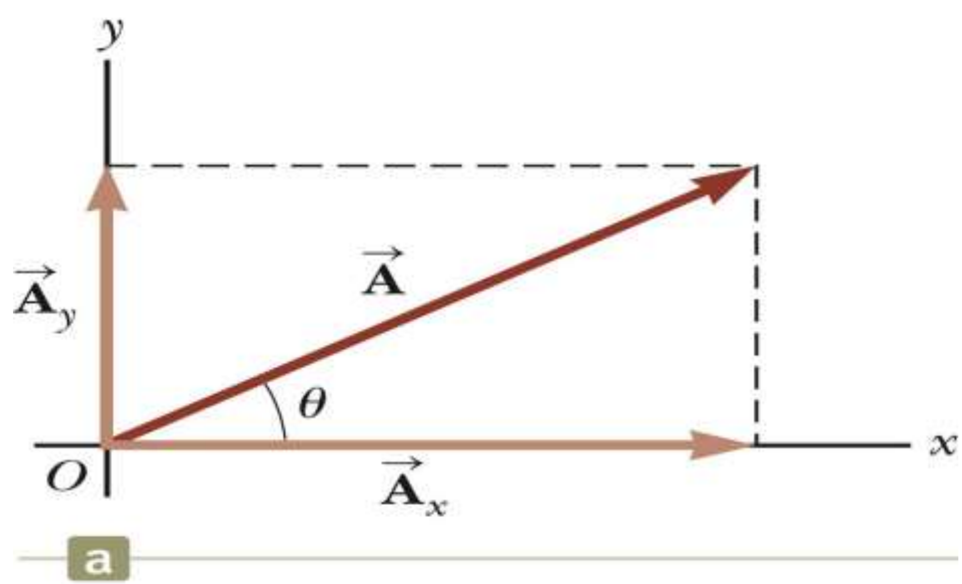
- The magnitude of the vector is multiplied or divided by the scalar, but the direction is same for the original and the new vector

- If the scalar is positive, **the direction of the result is the same as** of the original vector

- If the scalar is negative, the direction of the result is opposite that of the original vector

Components of a Vector

- A **component** is a projection of a vector along an axis.
- Any vector can be completely described by its **components**.
- It is useful to use rectangular components
- These are the projections of the vector along the x- and y-axes



Components of a Vector

$\mathbf{A}_x$ and $\mathbf{A}_y$ are the ***component vectors*** of **A**

They are vectors and follow all the rules for vectors

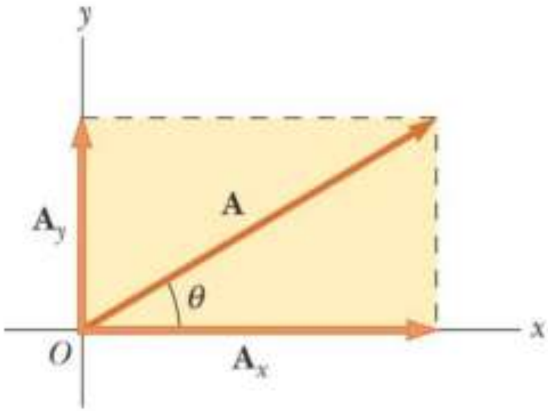
A_x and A_y are scalars, and will be referred to as the ***components*** of **A**

- The x-component of a vector is the projection along the x-axis

$$A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

$$A_y = A\sin\theta$$

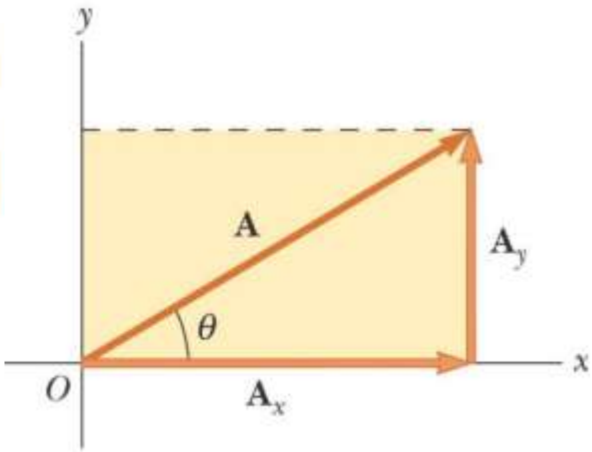


$\cos\theta = A_x/A$

$\sin\theta = A_y/A$

Components of a Vector

- The *y*-component is moved to the end of the *x*-component. This is due to the fact that any vector can be moved parallel to itself without being affected
  - This completes the triangle

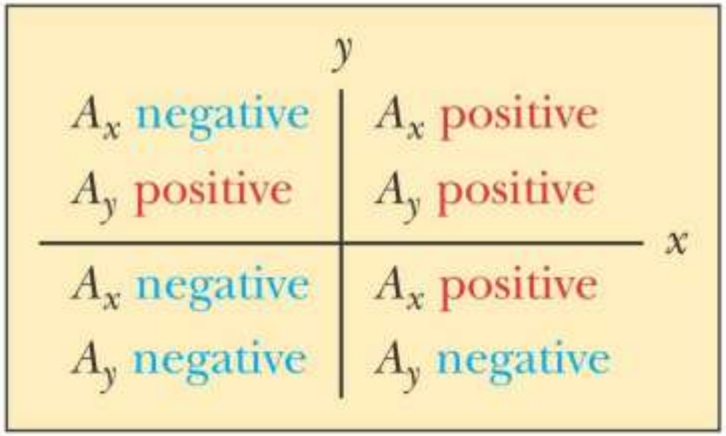


- *only if θ is taken with respect to the x-axis*

$$A = \sqrt{A_x^2 + A_y^2} \quad \text{and} \quad \theta = \tan^{-1}\frac{A_y}{A_x}$$

Components of a Vector

- The components can be positive or negative and will have the same units as the original vector
- The signs of the components will depend on the angle



Unit Vectors

A **unit vector** is a dimensionless vector with a magnitude of exactly 1.

Unit vectors are used to specify a direction and have no other physical significance.

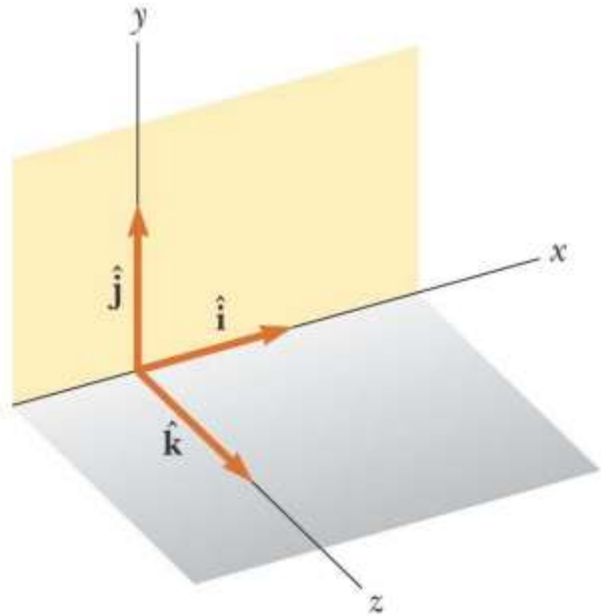
Unit Vectors, cont.

- The symbols

$$\hat{i}, \hat{j}, \text{and } \hat{k}$$

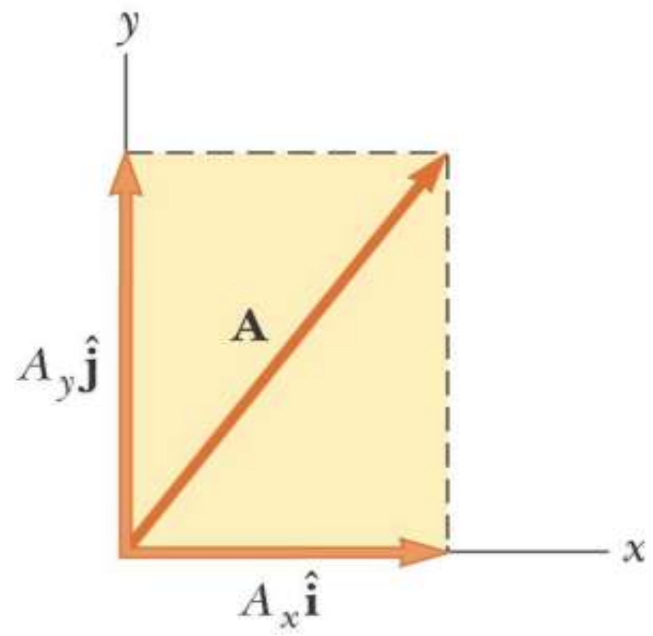
represent unit vectors

- They form a set of mutually perpendicular vectors



Unit Vectors in Vector Notation

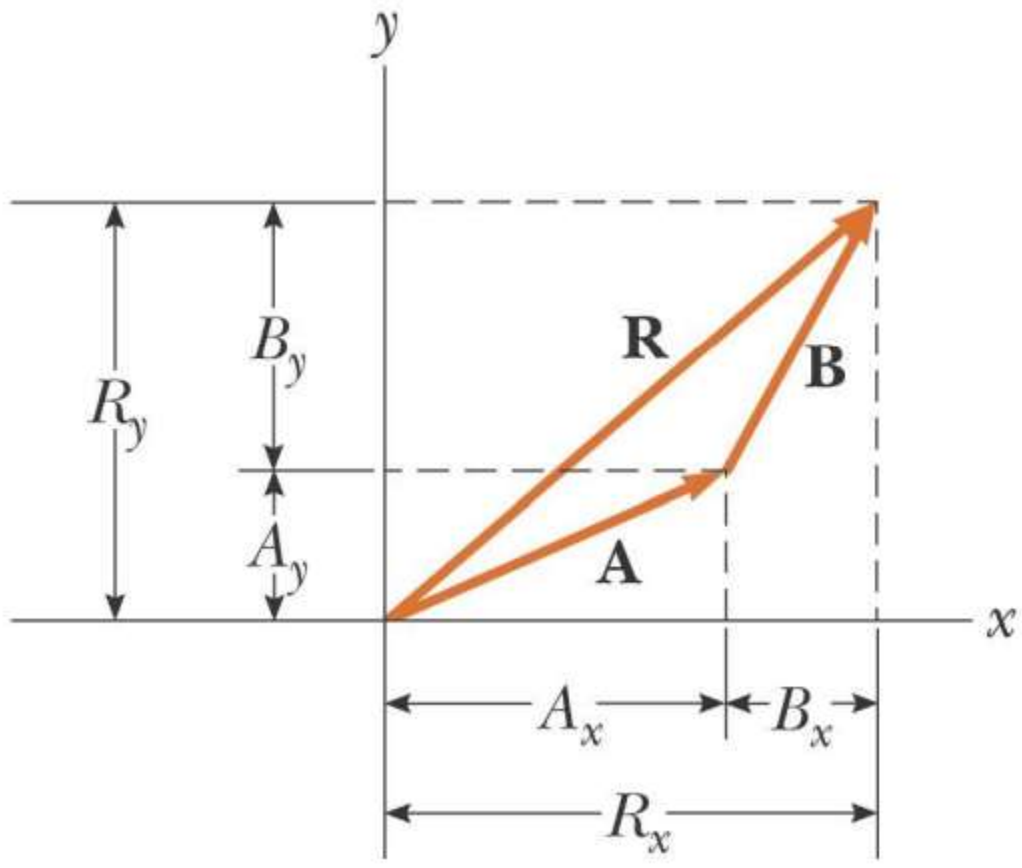
- $\mathbf{A_x}$ is the same as $A_x\hat{\imath}$ and
  $\mathbf{A_y}$ is the same as $A_y\hat{\jmath}$ and
  $\mathbf{A_z}$ is the same as $A_z\hat{k}$



The complete vector can be expressed as

$$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

Adding Vectors with Unit Vectors



Adding Vectors with Unit Vectors

- Using **R** = **A** + **B**
- Then

$$\mathbf{R} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\mathbf{R} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\mathbf{R} = R_x + R_y$$

- and so $R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Example 3.3 The Sum of Two Vectors

Find the sum of two displacement vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ lying in the xy plane and given by

$$\vec{\mathbf{A}} = (2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ m} \quad \text{and} \quad \vec{\mathbf{B}} = (2.0\hat{\mathbf{i}} - 4.0\hat{\mathbf{j}})\text{ m}$$

SOLUTION

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (2.0 + 2.0)\hat{\mathbf{i}}\text{ m} + (2.0 - 4.0)\hat{\mathbf{j}}\text{ m}$$

$$R_x = 4.0\text{ m} \qquad R_y = -2.0\text{ m}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0\text{ m})^2 + (-2.0\text{ m})^2} = \sqrt{20}\text{ m} = 4.5\text{ m}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\text{ m}}{4.0\text{ m}} = -0.50$$

Your calculator likely gives the answer $-27°$ for $\theta = \tan^{-1}(-0.50)$. This answer is correct if we interpret it to mean 27° clockwise from the x axis. Our standard form has been to quote the angles measured counterclockwise from the $+x$ axis, and that angle for this vector is $\theta =$ 333°.

Example 3.4 The Resultant Displacement

A particle undergoes three consecutive displacements: $\Delta\vec{\mathbf{r}}_1 = (15\hat{\mathbf{i}} + 30\hat{\mathbf{j}} + 12\hat{\mathbf{k}})$ cm, $\Delta\vec{\mathbf{r}}_2 = (23\hat{\mathbf{i}} - 14\hat{\mathbf{j}} - 5.0\hat{\mathbf{k}})$ cm, and $\Delta\vec{\mathbf{r}}_3 = (-13\hat{\mathbf{i}} + 15\hat{\mathbf{j}})$ cm. Find unit-vector notation for the resultant displacement and its magnitude.

SOLUTION

$$\Delta\vec{\mathbf{r}} = \Delta\vec{\mathbf{r}}_1 + \Delta\vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_3$$

$$= (15 + 23 - 13)\hat{\mathbf{i}}\text{ cm} + (30 - 14 + 15)\hat{\mathbf{j}}\text{ cm} + (12 - 5.0 + 0)\hat{\mathbf{k}}\text{ cm}$$

$$= (25\hat{\mathbf{i}} + 31\hat{\mathbf{j}} + 7.0\hat{\mathbf{k}})\text{ cm}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

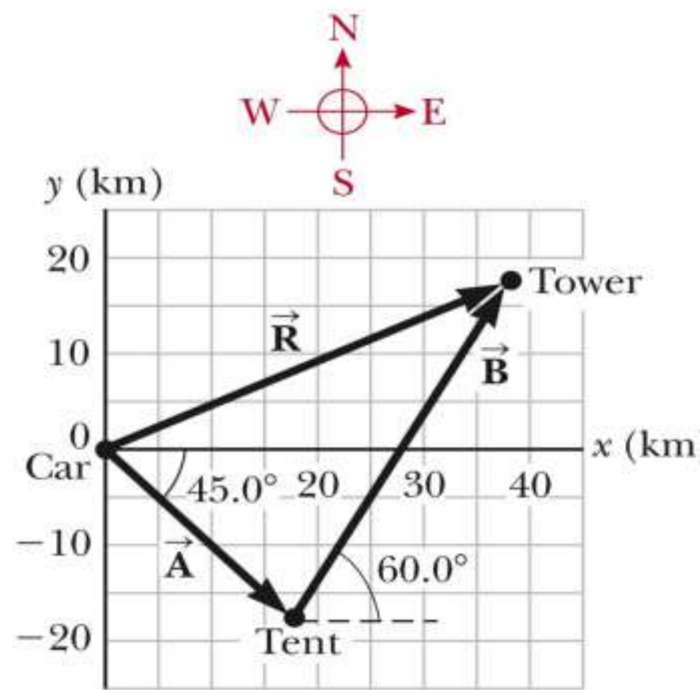
$$= \sqrt{(25\text{ cm})^2 + (31\text{ cm})^2 + (7.0\text{ cm})^2} = 40\text{ cm}$$

Example 3.5 – Taking a Hike

A hiker begins a trip by first walking 25.0 km southeast from her car. She stops and sets up her tent for the night. On the second day, she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

Example 3.5 – Solution, Conceptualize and Categorize

- Conceptualize the problem by drawing a sketch as in the figure.
- Denote the displacement vectors on the first and second days by $\vec{A}$ and $\vec{B}$ respectively.
- Use the car as the origin of coordinates.
- The vectors are shown in the figure.
- Drawing the resultant $\vec{R}$, we can now categorize this problem as an addition of two vectors.



Example 3.5 – Solution, Analysis

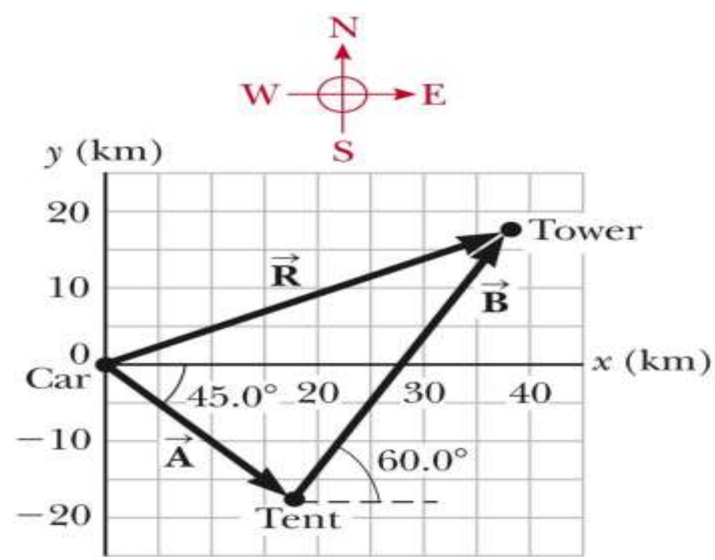
Analyze this problem by using our new knowledge of vector components. The first displacement has a magnitude of 25.0 km and is directed 45.0° below the positive x axis. Its components are:

A_x = A cos (45.0°)=

(25.0 km)(0.707) = 17.7 km

A_y = A sin (45.0°)=

(25.0 km)(-0.707) =- 17.7 km



Example 3.5 – Solution, Analysis 2

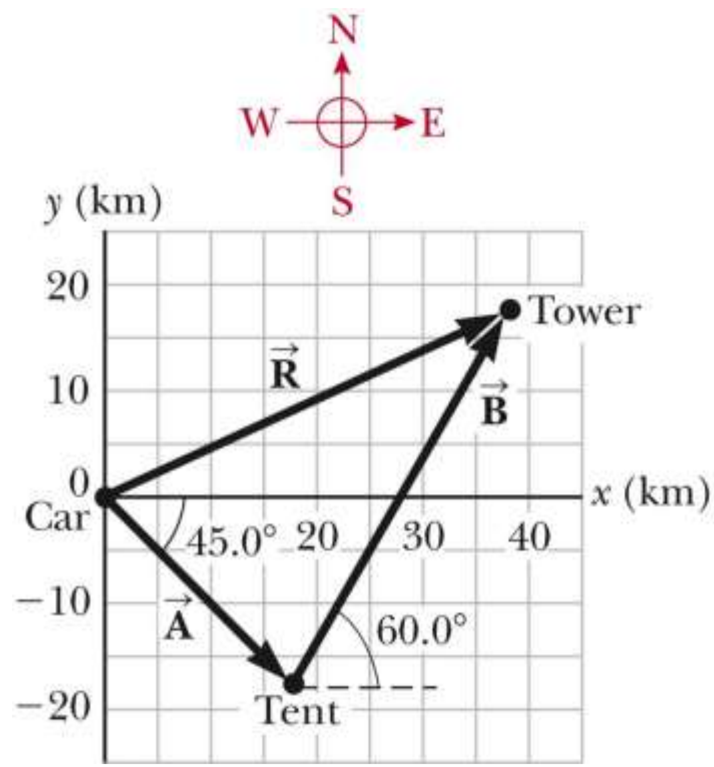
The second displacement has a magnitude of 40.0 km and is 60.0° north of east. Its components are:

$B_x = B\cos(60.0°) =$

$(40.0\text{ km})(0.500) = 20.0\text{ km}$

$B_y = B\sin(60.0°) =$

$(40.0\text{ km})(0.866) = 34.6\text{ km}$



Example 3.5 – Solution, Analysis 3

The negative value of A_y indicates that the hiker walks in the negative y direction on the first day. The signs of A_x and A_y also are evident from the figure. The signs of the components of B are also confirmed by the diagram.

Example 3.5 – Solution, Analysis

Determine the components of the hiker's resultant displacement for the trip.

- Find an expression for the resultant in terms of unit vectors. The resultant displacement for the trip has components given by

$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km}$

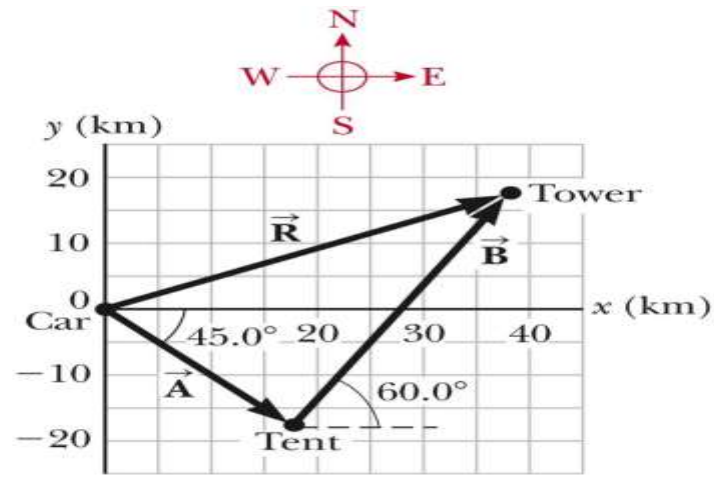
$= 37.7\text{ km}$

$R_y = A_y + B_y = -17.7\text{ km} + 34.6\text{ km}$

$= 16.9\text{ km}$

In unit vector form

$\vec{R} = (37.7\,\hat{\imath} + 16.9\,\hat{\jmath})\text{ km}$

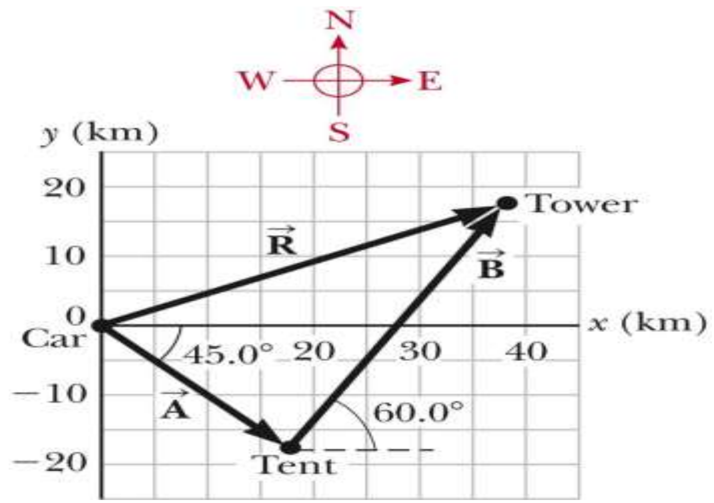


Example 3.5 – Solution, Finalize

The resultant vector has a magnitude of 41.3 km and is directed 24.1° north of east.

The units of $\vec{R}$ are km, which is reasonable for a displacement.

From the graphical representation , estimate that the final position of the hiker is at about (38 km, 17 km) which is consistent with the components of the resultant.



PROBLEMS

Section 3.1 Coordinate Systems

1. The polar coordinates of a point are $r = 5.50$ m and $\theta = 240°$. What are the Cartesian coordinates of this point?

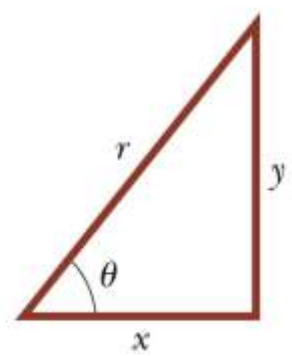
SOLUTIONS TO PROBLEM:

- $x = r \cos\theta$
- $y = r \sin\theta$

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x}$$



PROBLEMS

- **Section 3.1 Coordinate Systems**

4. Two points in the *xy* plane have Cartesian coordinates (2.00, -4.00) m and (-3.00, 3.00) m. Determine (a) the distance between these points and (b) their polar coordinates.

SOLUTIONS TO PROBLEM:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

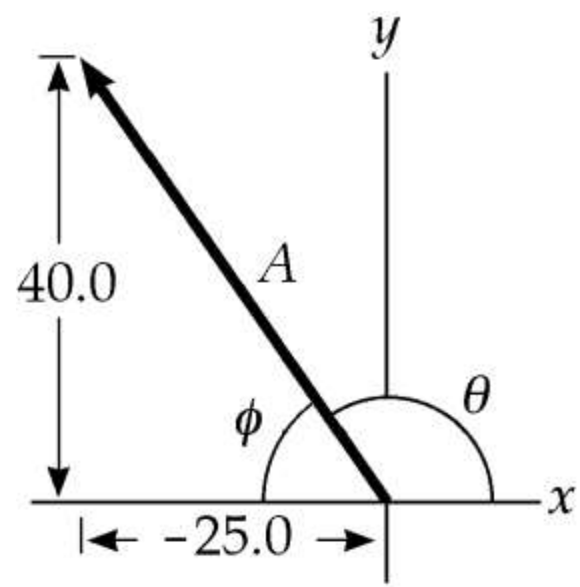
$$r_1 = \sqrt{{x_1}^2 + {y_1}^2} \text{ and } r_2 = \sqrt{{x_2}^2 + {y_2}^2}$$

$$\theta_1 = tan^{-1}\frac{y_1}{x_1} \text{ and } \theta_2 = tan^{-1}\frac{y_2}{x_2}$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

19. A vector has an *x* component of -25.0 units and a *y* component of 40.0 units. Find the magnitude and direction of this vector.



SOLUTIONS TO PROBLEM:

$$A = \sqrt{A_x{}^2 + A_y{}^2}$$

$$\tan\emptyset = \frac{|A_y|}{|A_x|} \rightarrow \emptyset = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right)$$

$$\theta = 180 - \emptyset$$

PROBLEMS

Section 3.4 Components of a Vector and Unit Vectors

21. Obtain expressions in component form for the position vectors having the following polar coordinates: (a) 12.8 m, 150° (b) 3.30 cm, 60.0° (c) 22.0 in., 215°.

SOLUTIONS TO PROBLEM:

$x = r \cos \theta$ and $y = r \sin \theta$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

27. Given the vectors

$$\boldsymbol{A}=2.00\,\hat{\imath}+6.00\hat{\jmath}$$

And

$$\mathbf{B}=3.00\,\hat{\imath}+2.00\hat{\jmath}$$

(a) draw the vector sum **C** = **A** + **B** and the vector difference **D** = **A** - **B**.
(b) Calculate **C** and **D**, first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the + *x* axis.

SOLUTIONS TO PROBLEM:

$$\boldsymbol{C}=(A_x+B_x)\hat{\imath}+(A_y+B_y)\hat{\jmath}$$
$$\boldsymbol{D}=(A_x-B_x)\hat{\imath}+(A_y-B_y)\hat{\jmath}$$

$C_x=A_x+B_x$ and $C_y=A_y+B_y$ C= $\sqrt{C_x{}^2+C_y{}^2}$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

30. Vector **A** has *x* and *y* components of -8.70 cm and 15.0 cm, respectively; vector **B** has *x* and *y* components of 13.2 cm and -6.60 cm, respectively. If **A** - **B** +3**C** =0, what are the components of **C**?

SOLUTIONS TO PROBLEM:

$C_x = A_x + B_x$ and $C_y = A_y + B_y$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

31. Consider the two vectors $\boldsymbol{A}$=3 $\hat{\imath} - 2\hat{\jmath}$ and **B**=$- \hat{\imath} - 4\hat{\jmath}$.

Calculate

(a) A + B,

(b) **A - B,**

(c) | **A + B** |,

(d) | **A - B** |,

and (e) the directions of **A + B** and **A – B**.

SOLUTIONS TO PROBLEM:

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$$
$$\mathbf{A} - \mathbf{B} = (A_x - B_x)\hat{\imath} + (A_y - B_y)\hat{\jmath}$$

$$|\mathbf{A} + \mathbf{B}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

33. A particle undergoes the following consecutive displacements: 3.50 m south, 8.20 m northeast, and 15.0 m west.

What is the resultant displacement?

SOLUTIONS TO PROBLEM:

$$\boldsymbol{d_1 = (-3.5\hat{j})m}$$
$$\boldsymbol{d_2 = (8.20\ cos45\hat{\imath} + 8.20\ sin45)m}$$
$$\boldsymbol{d_1 = (-15\hat{\imath})m}$$
$$\boldsymbol{R = d_1 + d_2 + d_3}$$

The magnitude of the resultant displacement is R $= |\boldsymbol{R}| = \sqrt{R_x{}^2 + R_y{}^2}$ **and** The direction is

$\theta = \boldsymbol{tan^{-1}}\left(\frac{R_y}{R_x}\right)$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

39. Vector **B** has *x*, *y*, and *z* components of 4.00, 6.00, and 3.00 units, respectively. Calculate the magnitude of **B** and the angles that **B** makes with the coordinate axes.

SOLUTIONS TO PROBLEM:

$$\mathbf{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

$$|\boldsymbol{B}| = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2}$$

$$\alpha = \cos^{-1}\frac{\mathbf{B_x}}{|\boldsymbol{B}|};\ \beta = \cos^{-1}\frac{\mathbf{B}_y}{|\boldsymbol{B}|}\ \textit{and}\ \gamma = \cos^{-1}\frac{\mathbf{B}_z}{|\boldsymbol{B}|}$$

PROBLEMS

- **Section 3.4 Components of a Vector and Unit Vectors**

49. Three displacement vectors of a croquet ball are shown in Figure P3.49, where |**A**|= 20.0 units, |**B**| =40.0 units, and |**C**|=30.0 units. Find (a) the resultant in unit–vector notation and (b) the magnitude and direction of the resultant displacement.

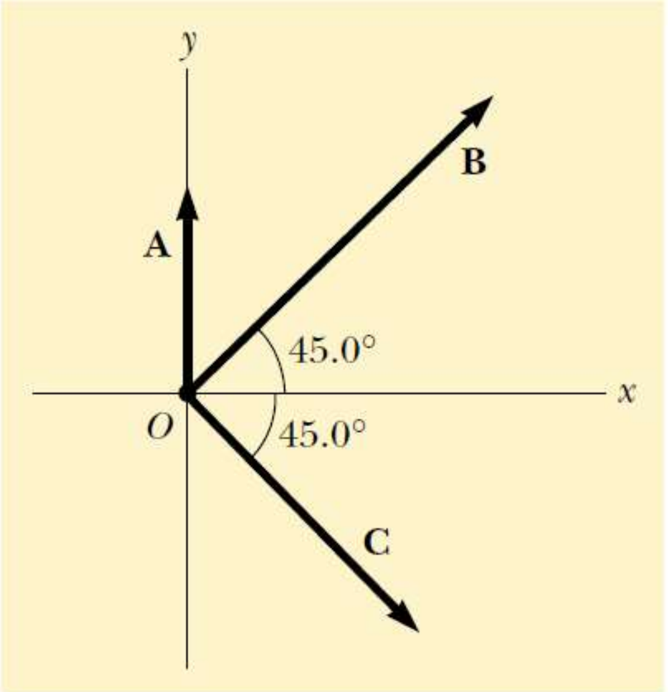


Figure P3.49

SOLUTIONS TO PROBLEM:

$$R_x = (40\ cos45\hat{\imath} + 30\ cos45)$$
$$R_y = (40\ sin45\hat{\imath} - 30\ sin45)$$

$$R = |\boldsymbol{R}| = \sqrt{{R_x}^2 + {R_y}^2}\ \textbf{and}\ \theta = \boldsymbol{tan^{-1}}\left(\frac{R_y}{R_x}\right)$$

PROBLEMS

Section 3.4 Components of a Vector and Unit Vectors

50. If **A** = (6.00ˆ**i** - 8.00ˆ**j**) units, **B** = (-8.00ˆ**i** + 3.00ˆ**j**) units, and **C** = (2

6.0ˆ**i** +19.0ˆ**j**) units, determine a and b such that $a\mathbf{A} + b\mathbf{B} + \mathbf{C} = 0$.

SOLUTIONS TO PROBLEM:

Taking components along ˆ**i** and ˆ**j** , we get two equations:

$6.00a - 8.00b + 26.0 = 0$ and $-8.00a + 3.00b + 19.0 = 0$.

Solving simultaneously,

$a = 5.00$, $b = 7.00$.

Therefore, $5.00\mathbf{A} + 7.00\mathbf{B} + \mathbf{C} = 0$.