Phys 103
Chapter 6
Circular Motion and Other Applications of Newton's Laws

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LECTURE OUTLINE

6.1 Newton’s Second Law Applied to Uniform Circular Motion

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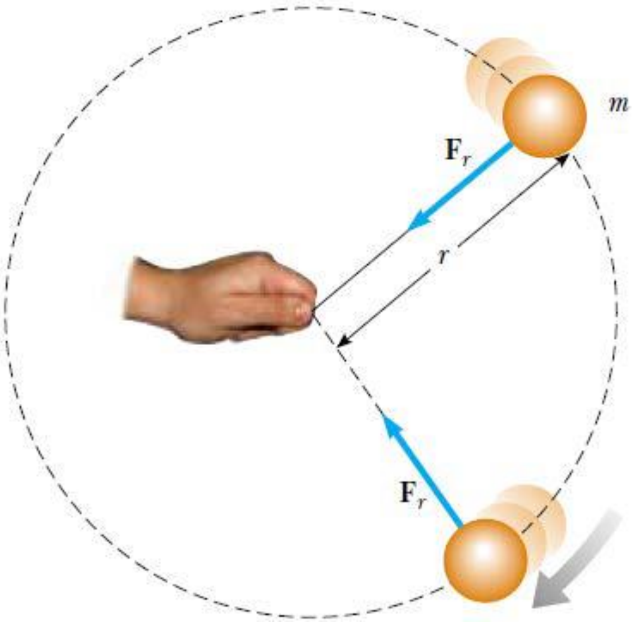
- ***Newton's Second Law***

Applying Newton's Second Law to Uniform Circular Motion we get:

$$a_c = \frac{v^2}{r}$$

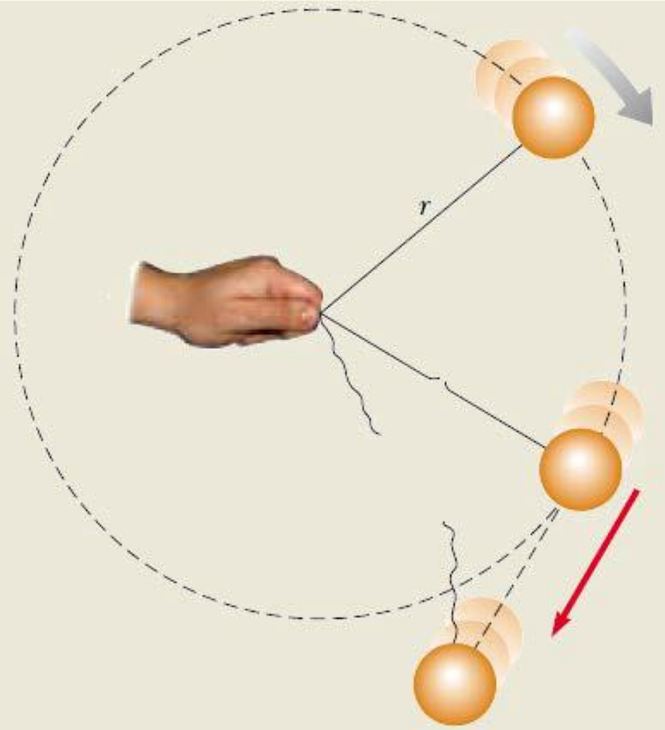
The acceleration is called centripetal acceleration because ac is directed toward the center of the circle **a**$_c$ is always perpendicular to **v.** If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:



$$\sum \boldsymbol{F} = \boldsymbol{m}\boldsymbol{a}_c = \boldsymbol{m}\frac{v^2}{r}$$

6.1 Newton's Second Law Applied to Uniform Circular Motion

Circular Motion



A force causing a *centripetal acceleration* acts toward the *center of the circular path* and causes a change in the direction of the velocity vector.

If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle.

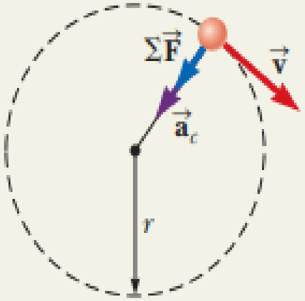
This idea is illustrated in the figure for the ball whirling at the end of a string in a horizontal plane. If the string breaks at some instant, the ball moves along the straight-line path tangent to the circle at the point where the string breaks.

Analysis Model Particle in Uniform Circular Motion (Extension)

Imagine a moving object that can be modeled as a particle. If it moves in a circular path of radius r at a constant speed v, it experiences a centripetal acceleration. Because the particle is accelerating, there must be a net force acting on the particle. That force is directed toward the center of the circular path and is given by

$$\sum F = ma_c = m\frac{v^2}{r} \quad \textbf{(6.1)}$$

Examples



- the tension in a string of constant length acting on a rock twirled in a circle
- the gravitational force acting on a planet traveling around the Sun in a perfectly circular orbit (Chapter 13)
- the magnetic force acting on a charged particle moving in a uniform magnetic field (Chapter 29)
- the electric force acting on an electron in orbit around a nucleus in the Bohr model of the hydrogen atom (Chapter 42)

6.1 Newton's Second Law Applied to Uniform Circular Motion

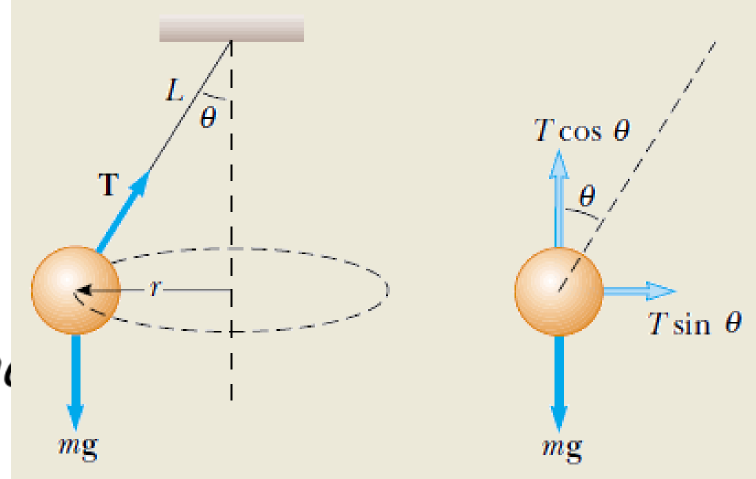
Example 6.2 The Conical Pendulum

A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in Figure. (Because the string sweeps out the surface of a cone, the system is known as a conical pendulum.) *Find an expression for v.*

Solution:

We shall apply Newton's 2nd law as we did before.

We need first to analyze forces and apply the law in x, then y directions. $\sum F_x = \boldsymbol{m}\boldsymbol{a}_x$



$\sum F_x = ma_x$

6.1 Newton's Second Law Applied to Uniform Circular Motion

- ***Example 6.2 The Conical Pendulum (continued)***

$$T\cos\theta = mg$$

$$T\sin\theta = ma_c = m\frac{v^2}{r}$$

$$\frac{T\sin\theta}{T\cos\theta} = \frac{m\frac{v^2}{r}}{mg} \Rightarrow \tan\theta = \frac{v^2}{gr}$$

$$v = \sqrt{gr\tan\theta}$$

$$r = L\sin\theta \Rightarrow v = \sqrt{Lg\sin\theta\tan\theta}$$

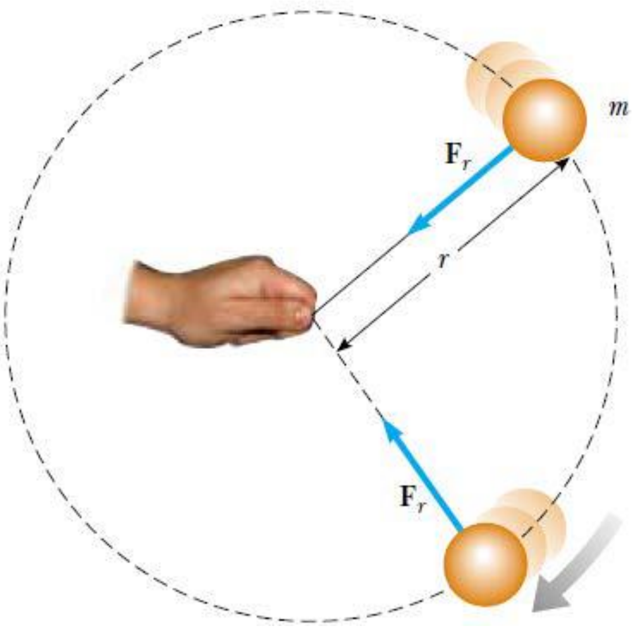
Example 6.2 How Fast Can It Spin? AM

A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in Figure 6.1. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the puck can move before the cord breaks? Assume the string remains horizontal during the motion.

$$T = m\frac{v^2}{r}$$

$$(1) \quad v = \sqrt{\frac{Tr}{m}}$$

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0\text{ N})(1.50\text{ m})}{0.500\text{ kg}}} = \boxed{12.2\text{ m/s}}$$



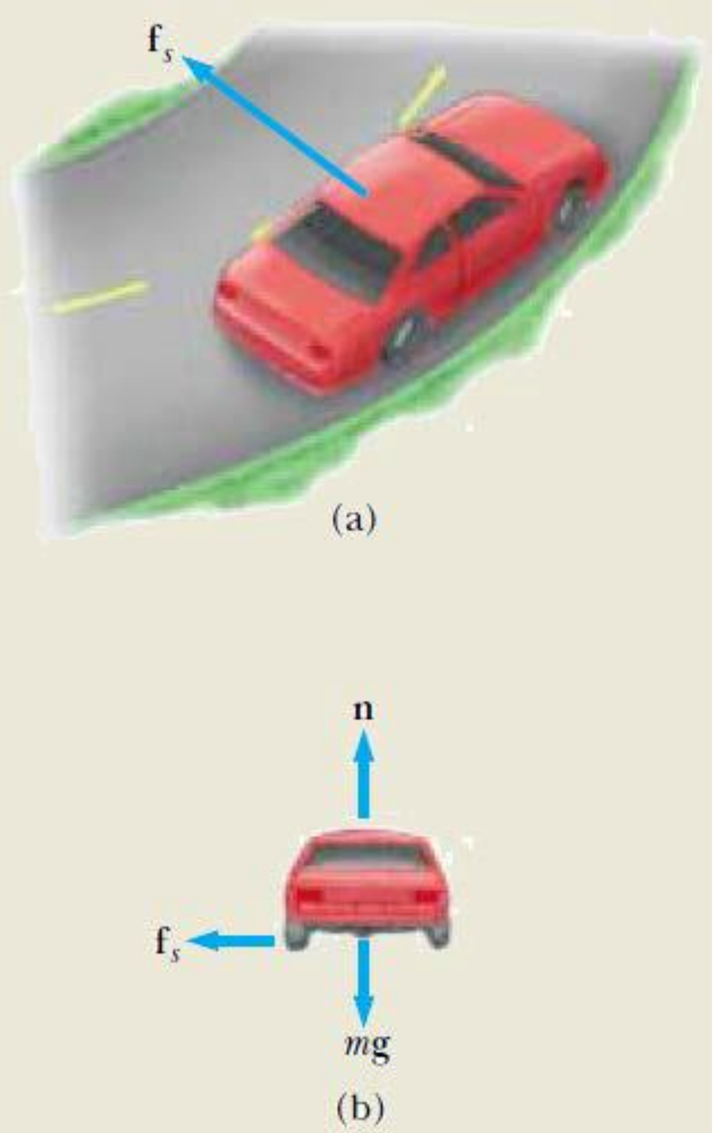
6.1 Newton's Second Law Applied to Uniform Circular Motion

Example 6.4 A car on a Curve

A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500*, find the maximum speed the car can have and still make the turn successfully.*

***Solution*: In this case, the force that enables the car to remain in its circular path is the force of *static friction*. (Static because no slipping occurs at the point of contact between road and tires.

- We shall apply Newton's 2nd law.



6.1 Newton's Second Law Applied to Uniform Circular Motion

- ***Example 6.4 A car on a Curve (continued)***

- Solving we get:

$$\sum F_x = \boldsymbol{ma}_x$$

$$f_s = \boldsymbol{m}\frac{v^2}{r}$$

$$f_s = \mu_s n = \mu_s mg$$

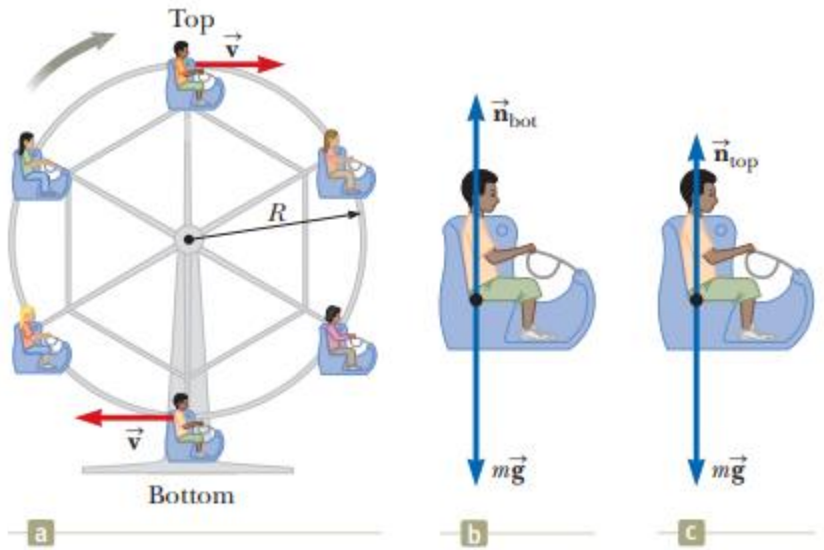
$$\mu_s mg = \boldsymbol{m}\frac{v^2}{r}$$

$$v = \sqrt{r\mu_s g} = \sqrt{(35)(0.5)(9.8)} = 13.1\ m/s$$

Example 6.5 Riding the Ferris Wheel AM

A child of mass m rides on a Ferris wheel as shown in Figure 6.6a. The child moves in a vertical circle of radius 10.0 m at a constant speed of 3.00 m/s.

(A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, mg.



$$\sum F = n_{bot} - mg = m\frac{v^2}{r}$$

$$n_{bot} = mg + m\frac{v^2}{r} = mg\left(1 + \frac{v^2}{rg}\right)$$

$$n_{bot} = mg\left[1 + \frac{(3.00\text{ m/s})^2}{(10.0\text{ m})(9.80\text{ m/s}^2)}\right]$$

$$= \boxed{1.09\,mg}$$

(B) Determine the force exerted by the seat on the child at the top of the ride.

$$\sum F = mg - n_{\text{top}} = m\frac{v^2}{r}$$

$$n_{\text{top}} = mg - m\frac{v^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$$

$$n_{\text{top}} = mg\left[1 - \frac{(3.00 \text{ m/s})^2}{(10.0 \text{ m})(9.80 \text{ m/s}^2)}\right]$$

$$= \boxed{0.908\, mg}$$

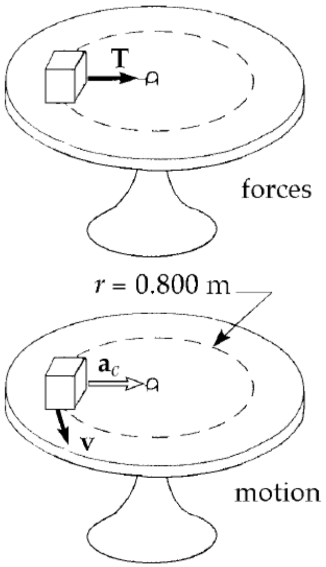
PROBLEMS

Sections 6.1 Newton's Second Law Applied to Uniform Circular Motion

1. A light string can support a stationary hanging load of 25.0 kg before breaking. A 3.00-kg object attached to the string rotates on a horizontal, frictionless table in a circle of radius 0.800 m, while the other end of the string is held fixed. What range of speeds can the object have before the string breaks?

SOLUTIONS TO PROBLEM:

- m = 3.00 kg , r = 0.800 m. The string will break if the te the weight corresponding to 25.0 kg, so $T_{max} = Mg$
- When the 3.00 kg mass rotates in a horizontal circle, th causes the centripetal acceleration, $\mathrm{T} = \boldsymbol{m}\frac{v^2}{r}$



PROBLEMS

Sections 6.1 Newton's Second Law Applied to Uniform Circular Motion

2. A curve in a road forms part of a horizontal circle. As a car goes around it at constant speed 14.0 m/s, the total force on the driver has magnitude 130 N. What is the total vector force on the driver if the speed is 18.0 m/s instead?

SOLUTIONS TO PROBLEM:

In $\sum F_x = \boldsymbol{m}\frac{v^2}{r}$, both *m* and *r* are unknown but remain constant. Therefore, ΣF is proportional to v^2 and increases by a factor of $(\frac{18}{14})^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then $\sum F_{fast} = \left(\frac{18}{14}\right)^2 (130)$Symbolically, write $\sum F_{slow}$ $= \frac{m}{r}(14)^2$ and $\sum F_{fast} = \frac{m}{r}(18)^2$ Dividing gives $\frac{\sum F_{fast}}{\sum F_{slow}} = \left(\frac{18}{14}\right)^2$

PROBLEMS

Sections 6.1 Newton's Second Law Applied to Uniform Circular Motion

5. A coin placed 30.0 cm from the center of a rotating, horizontal turntable slips when its speed is 50.0 cm/s. (a) What force causes the centripetal acceleration when the coin is stationary relative to the turntable? (b) What is the coefficient of static friction between coin and turntable?

SOLUTIONS TO PROBLEM:

static friction

$ma\mathrm{i} = f\,\mathrm{i} + n\,\mathrm{j} + mg\,(-\mathrm{j})$

$\sum F_y = 0 = n - mg$ thus $n = mg$ and

$$\sum F_r = m\frac{v^2}{r} = f = \mu\, n = \mu\, mg$$

Then $\mu = \frac{v^2}{rg}$

PROBLEMS

Sections 6.1 Newton's Second Law Applied to Uniform Circular Motion

7. A crate of eggs is located in the middle of the flat bed of a pickup truck as the truck negotiates an unbanked curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.600, how fast can the truck be moving without the crate sliding?

SOLUTIONS TO PROBLEM:

$n = mg$ since $a_y = 0$

The force causing the centripetal acceleration is the frictional force f.

From Newton's second law $f = ma_c = \boldsymbol{m}\frac{v^2}{r}$ But the friction condition is $f_s \leq \mu_s n$

$$\boldsymbol{m}\frac{v^2}{r} \leq \mu_s mg$$

PROBLEMS

Sections 6.1 Newton's Second Law Applied to Uniform Circular Motion

59. The pilot of an airplane executes a constant-speed loop-theloop maneuver in a vertical circle. The speed of the airplane is 300 mi/h, and the radius of the circle is 1 200 ft. (a) What is the pilot's apparent weight at the lowest point if his true weight is 160 lb? (b) What is his apparent weight at the highest point? (c) **What If?** Describe how the pilot could experience weightlessness if both the radius and the speed can be varied. (*Note:* His apparent weight is equal to the magnitude of the force exerted by the seat on his body.)

SOLUTIONS TO PROBLEM:

At the lowest point, his seat exerts an upward force; therefore, his weight seems to ... rent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1\,200} = \boxed{967\text{ lb}}.$$

At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = \boxed{-647\text{ lb}}.$$

When $F_g' = 0$, then $mg = \dfrac{mv^2}{R}$.

Lecture Summary

- **Newton's second law** applied to a particle moving in uniform circular motion states that the net force causing the particle to undergo a centripetal acceleration is:

$$\sum \boldsymbol{F} = \boldsymbol{m}\boldsymbol{a_c} = \boldsymbol{m}\frac{\boldsymbol{v^2}}{\boldsymbol{r}}$$

A particle moving in a uniform circular motion has the centripetal acceleration give by:

$$\boldsymbol{a_c} = \frac{v^2}{r}$$

(centripetal acceleration)