Phys 103
Chapter 8
Potential Energy

Dr. Wafa Almujamammi

LECTURE OUTLINE

- 8.1 Potential Energy of a System
- 8.2 The Isolated System—Conservation of Mechanical Energy
- 8.3 Conservative and Nonconservative Forces
- 8.4 Changes in Mechanical Energy for Nonconservative Forces
- 8.5 Relationship Between Conservative Forces and Potential Energy

Introduction

- In Chapter 7 we introduced the concepts of kinetic energy associated with the motion of members of a system and internal energy associated with the temperature of a system.
- In this chapter we introduce ***potential energy***, the energy associated with the configuration of a system of objects that exert forces on each other.
- The potential energy concept can be used only when dealing with a special class of forces called ***conservative forces***. When only conservative forces act within an isolated system,
- the kinetic energy gained (or lost) by the system as its members change their relative positions is balanced by an equal loss (or gain) in potential energy. This balancing of the two forms of energy is known as the ***principle of conservation of mechanical energy***.

8.1 Potential Energy of a System

- Let us now derive an expression for the **gravitational potential energy** (U_g) associated with an object (m) at a given location (y) above the surface of the Earth

$$U_g = mgy$$

- Mathematical description of the work done on a system that changes the gravitational potential energy of the system is give by:

$$W = \triangle U_g$$

- The gravitational potential energy depends only on the vertical height of the object above the surface of the Earth. The same amount of work must be done on an object–Earth system whether the object is lifted vertically from the Earth or is pushed starting from the same point up a frictionless incline, ending up at the same height

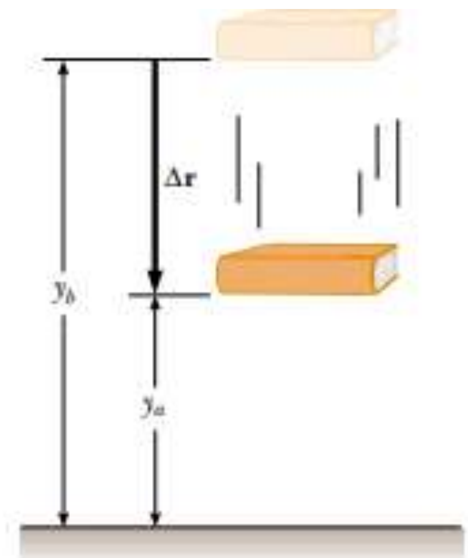
8.2 The Isolated System— Conservation of Mechanical Energy

- As the book, shown in the figure, falls back to its original height, from y_b to y_a, the work done by the gravitational force on the book is:

$$W = mgy_b - mgy_a = \triangle K = -\triangle U_g$$

So

$$\triangle K + \triangle U_g = 0$$



- Mechanical energy is defined as:

$$E_{mech} = K + U_g$$

- Or, in general:

$$E_{mech} = K + U$$

8.2 The Isolated System— Conservation of Mechanical Energy

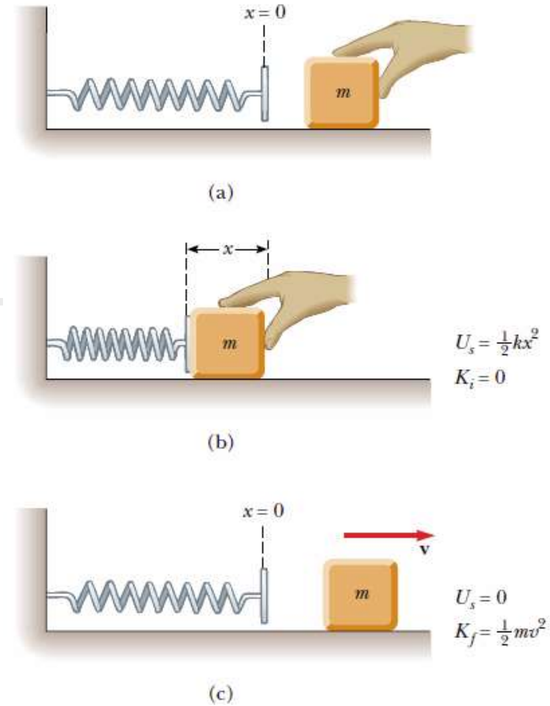
- Let us now write the changes in energy in Equation 8.7 explicitly:

$$(K_f - K_i) + (U_f - U_i) = 0$$
$$(K_f + U_f) - (K_i + U_i) = 0$$
$$K_f + U_f = K_i + U_i$$

- Equation 8.9 is a statement of conservation of mechanical energy for an isolated system.
- An isolated system is one for which there are no energy transfers across the boundary.
- The energy in such a system is conserved—the sum of the kinetic and
- Potential energies remains constant.
- This statement assumes that no nonconservative forces act within the system.

8.2 The Isolated System—Conservation of Mechanical Energy

Elastic Potential Energy



Potential Energy of a Spring

$$W_{F_{app}} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Potential Energy of a Spring is given by:

$$U_s = \frac{1}{2}kx^2$$

- When the block is released from rest, the spring exerts a force on the block and returns.
- to its original length. The stored elastic potential energy is transformed into kinetic energy of the block.
- The elastic potential energy stored in a spring is zero when: x = 0
- Energy is stored in the spring only when the spring is either stretched or compressed.

8.2 The Isolated System— Conservation of Mechanical Energy

Example 8.2 Ball in Free Fall

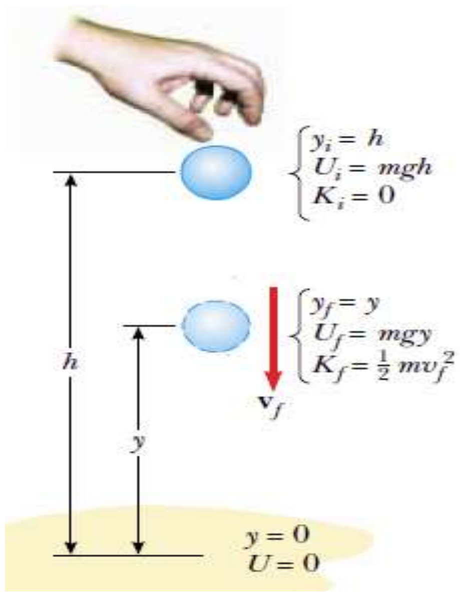
A ball of mass m is dropped from a height h above the ground, as shown in Figure . *Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground.*

- **Solution**:

$$K_f + U_f = K_i + U_f$$

$$\frac{1}{2}mv^2{}_f + mgy = 0 + mgh$$

$$v_f = \sqrt{2g(h-y)}$$



8.2 The Isolated System— Conservation of Mechanical Energy

- **Example 8.3 The Pendulum**

A pendulum consists of a sphere of mass m = 200 gm attached to a light cord of length L =50 cm, as shown in Figure. The sphere is released from rest at point A when the cord makes an angle θA=37owith the vertical.

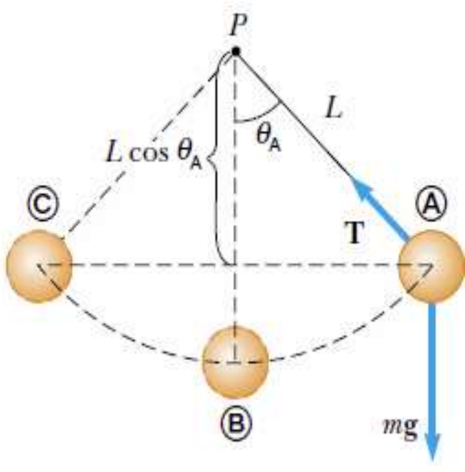
- (A) Find the speed of the sphere when it is at the lowest point B.
- **Solution**:

$$K_B + U_B = K_A + U_A$$

$$\frac{1}{2}mv^2{}_B - mgL = 0 - mgL\,cos\boldsymbol{\theta}_A$$

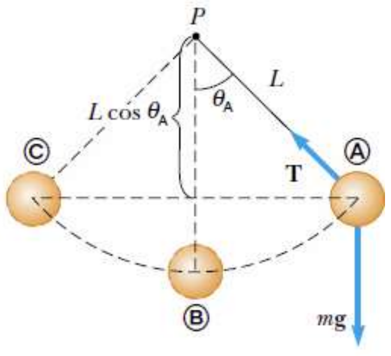
$$v_B = \sqrt{2gL(1 - cos\boldsymbol{\theta}_A)}$$

$$v_{B=1.4\,m/s}$$



8.2 The Isolated System— Conservation of Mechanical Energy

- **Example 8.3 The Pendulum**



 (B) What is the tension TB in the cord at B?

Solution:

Newton's second law gives:

$$\sum F_r = mg - T_B = ma_r = -m\frac{v^2{}_B}{L}$$

$$T_B = mg + m\frac{v^2{}_B}{L}$$

8.2 The Isolated System— Conservation of Mechanical Energy

Example 8.5 The Spring-Loaded Popgun

The launching mechanism of a toy gun consists of a spring of unknown spring constant. When the spring is compressed 0.120 m, the gun, when fired vertically, is able to launch a 35.0-g projectile to a maximum height of 20.0 m above the position of the projectile before firing.

(A) Neglecting all resistive forces, determine the spring constant.

Solution:

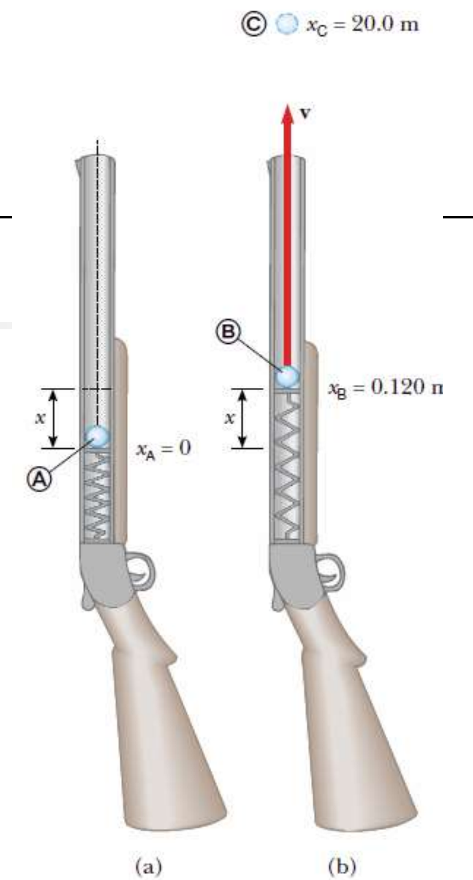
Total energy at position (c) for the projectile + spring = Total energy at position (A)

- Hence:

$$\left[K_{projectile} + U_{projectile} + U_{spring}\right]_C = \left[K_{projectile} + U_{projectile} + U_{spring}\right]_A$$

8.2 The Isolated System—Conservation of Mechanical Energy

Example 8.5 The Spring-Loaded Popgun



$$K_c + U_{gc} + U_{sc} = K_A + U_{gA} + U_{sA}$$

$$0 + mgh + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$k = \frac{2mgh}{x^2}$$

- Find the speed of the projectile as it moves through the equilibrium position of the spring at x_B

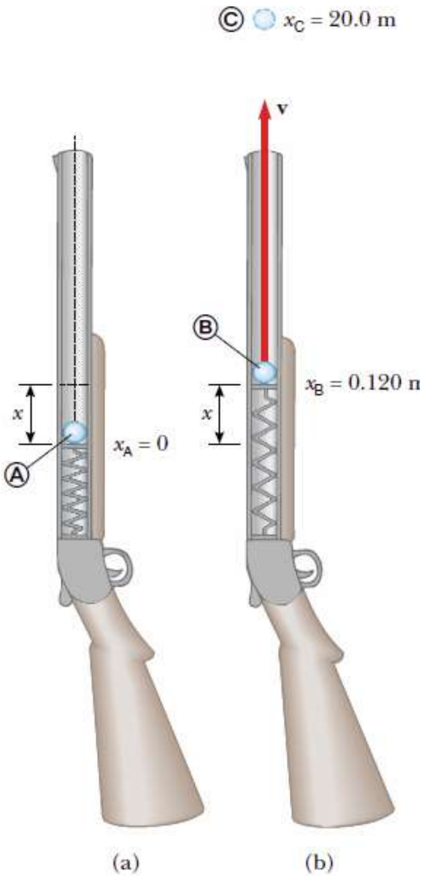
$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

$$\frac{1}{2}mv_B{}^2 + mgx_B + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$v_B = \sqrt{\frac{kx^2}{m} - 2gx_B}$$

$$= \sqrt{\frac{(953\ \text{N/m})(0.120\ \text{m})^2}{(0.0350\ \text{kg})} - 2(9.80\ \text{m/s}^2)(0.120\ \text{m})}$$

$$= \boxed{19.7\ \text{m/s}}$$



8.3 Conservative and Nonconservative Forces

Conservative Forces

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.)

Examples of Conservative Forces:

- 1.gravitational force
- 2.Spring force

8.4 Changes in Mechanical Energy for Nonconservative Forces

A force is nonconservative if it does not satisfy properties 1 and 2 for conservative forces.

Nonconservative forces acting within a system cause a change in the *mechanical energy* of the system.

As an example of the path dependence of the work, consider moving a book between two points on a table. If the book is moved in a straight

line along the path between points A and B; a certain amount of work against the kinetic friction force must be spent to keep the book moving at a constant speed.

Now, imagine that the book was pushed along a semicircular path. More work must have been performed against friction along this longer path than along the straight path.

Hence, The work done depends on the path, so the friction force cannot

- be conservative force.

8.4 Changes in Mechanical Energy for Nonconservative Forces

Consider a body sliding across a surface. As the body moves through a distance *d*, the only force that does work on it is the force of kinetic friction. This force causes a decrease in the kinetic energy of the body. This decrease was calculated in Chapter 7, leading to Equation 7.20, which we repeat here:

$$\Delta K = -f_k d$$

If there is also a change in potential energy then:

$$E_{mech} = \Delta K + \Delta U_g$$

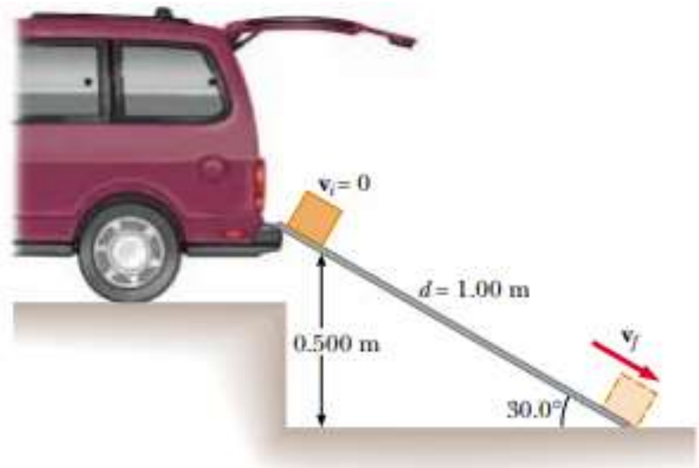
Or in general, for any potential:

$$E_{mech} = \Delta K + \Delta U = -f_k d$$

where ΔU is the change in all forms of potential energy.

Example 8.6 Crate Sliding Down a Ramp

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0°, as shown in Figure 8.11. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp. Use energy methods to determine the speed of the crate at the bottom of the ramp.



$$E_i = K_i + U_i = 0 + U_i = mgy_i$$
$$= (3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) = 14.7\text{ J}$$

$$E_f = K_f + U_f = \tfrac{1}{2}mv_f^2 + 0$$

$$-f_k d = (-5.00\text{ N})(1.00\text{ m}) = -5.00\text{ J}$$

$$E_f - E_i = \tfrac{1}{2}mv_f^2 - mgy_i = -f_k d$$
$$\tfrac{1}{2}mv_f^2 = 14.7\text{ J} - 5.00\text{ J} = 9.70\text{ J}$$
$$v_f^2 = \frac{19.4\text{ J}}{3.00\text{ kg}} = 6.47\text{ m}^2/\text{s}^2$$
$$v_f = 2.54\text{ m/s}$$

Example 8.7 Motion on a Curved Track

A child of mass m rides on an irregularly curved slide of height $h = 2.00$ m, as shown in Figure 8.12. The child starts from rest at the top.

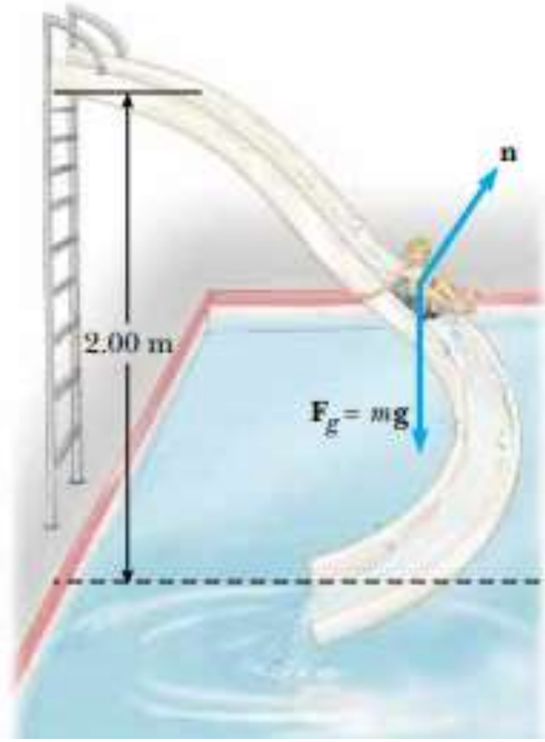
(A) Determine his speed at the bottom, assuming no friction is present.

$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2} m v_f^2 + 0 = 0 + mgh$$

$$v_f = \sqrt{2gh}$$

$$v_f = \sqrt{2gh} = \sqrt{2(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{m})} = 6.26\ \mathrm{m/s}$$

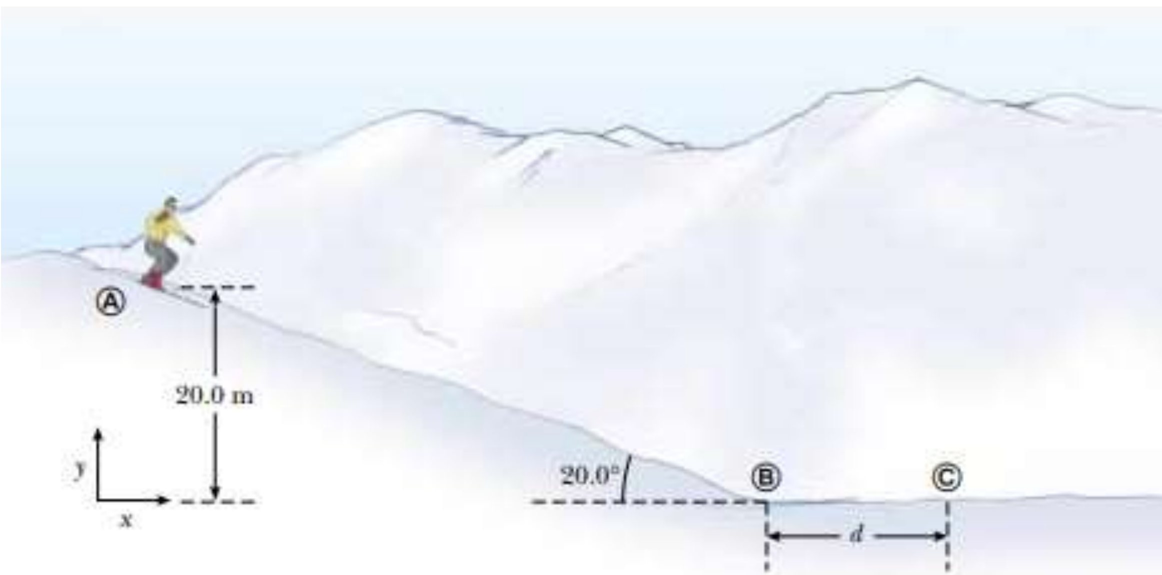


(B) If a force of kinetic friction acts on the child, how much mechanical energy does the system lose? Assume that $v_f = 3.00$ m/s and $m = 20.0$ kg.

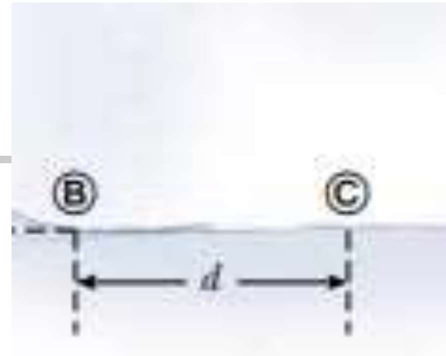
$$\Delta E_{\text{mech}} = (K_f + U_f) - (K_i + U_i)$$
$$= (\tfrac{1}{2} m v_f^2 + 0) - (0 + mgh) = \tfrac{1}{2} m v_f^2 - mgh$$
$$= \tfrac{1}{2}(20.0\text{ kg})(3.00\text{ m/s})^2$$
$$- (20.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})$$
$$= -302\text{ J}$$

Example 8.8 Let's Go Skiing!

A skier starts from rest at the top of a frictionless incline of height 20.0 m, as shown in Figure 8.13. At the bottom of the incline, she encounters a horizontal surface where the coefficient of kinetic friction between the skis and the snow is 0.210. How far does she travel on the horizontal surface before coming to rest, if she simply coasts to a stop?



$$v_B = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m})} = 19.8\ \text{m/s}$$



$f_k = \mu_k n = \mu_k mg$, we obtain

$$\Delta E_{mech} = E_C - E_B = -\mu_k mgd$$

$$(K_C + U_C) - (K_B + U_B) = (0 + 0) - (\tfrac{1}{2}mv_B^2 + 0) = -\mu_k mgd$$

$$d = \frac{v_B^2}{2\mu_k g} = \frac{(19.8\ \text{m/s})^2}{2(0.210)(9.80\ \text{m/s}^2)} = \boxed{95.2\ \text{m}}$$

Example 8.9 Block–Spring Collision

A block having a mass of 0.80 kg is given an initial velocity $v_A = 1.2\,\text{m/s}$ to the right and collides with a spring of negligible mass and force constant $k = 50\,\text{N/m}$, as shown in Figure 8.14.

(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

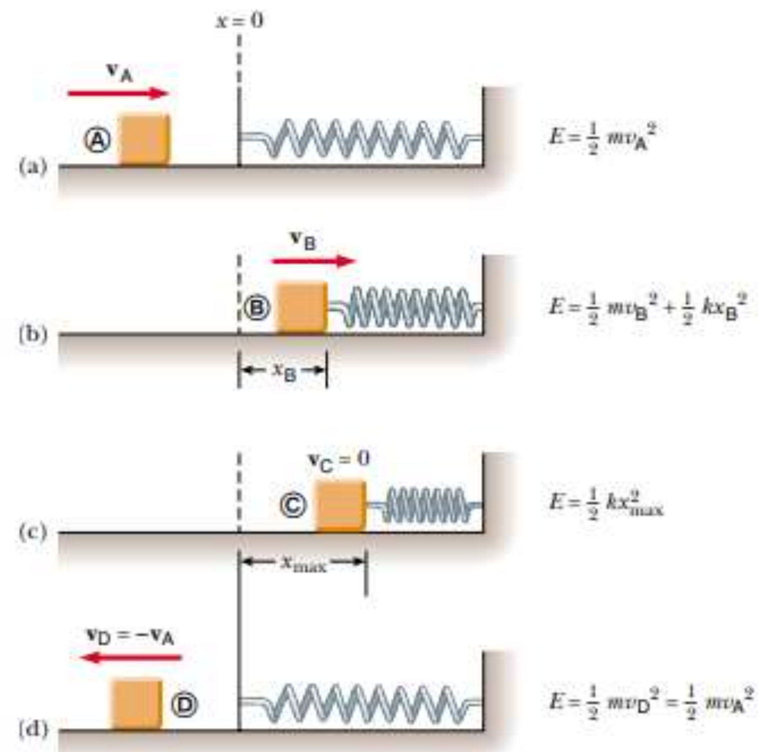
$$E_C = E_A$$

$$K_C + U_{sC} = K_A + U_{sA}$$

$$0 + \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}mv_A^2 + 0$$

$$x_{\max} = \sqrt{\frac{m}{k}}\,v_A = \sqrt{\frac{0.80\text{ kg}}{50\text{ N/m}}}\,(1.2\text{ m/s})$$

$$= 0.15\text{ m}$$



(B) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_A = 1.2$ m/s, what is the maximum compression x_C in the spring?

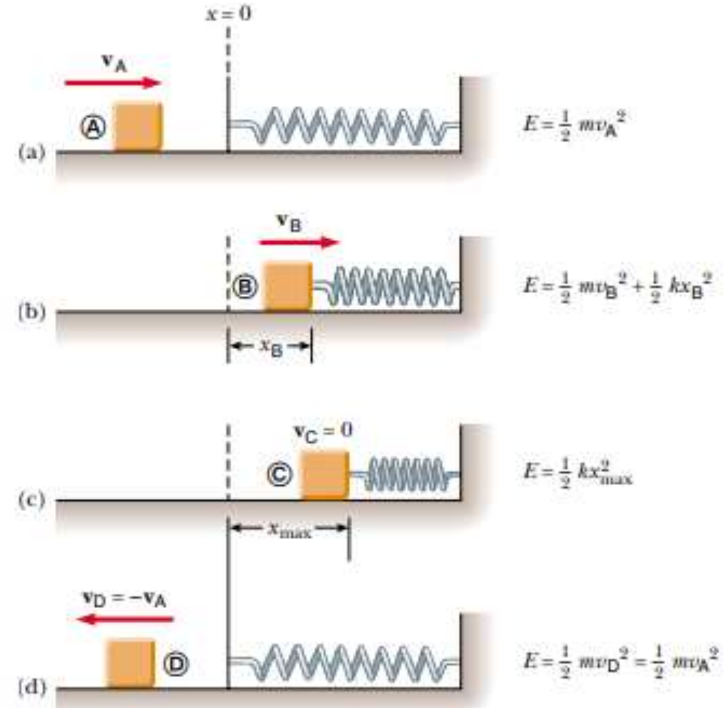
$$f_k = \mu_k n = \mu_k mg = 0.50(0.80\text{ kg})(9.80\text{ m/s}^2) = 3.92\text{ N}$$

$$\Delta E_{mech} = -f_k x_C = (-3.92 x_C)$$

$$\Delta E_{mech} = E_f - E_i = (0 + \tfrac{1}{2}k x_C{}^2) - (\tfrac{1}{2}mv_A{}^2 + 0) = -f_k x_C$$

$$\tfrac{1}{2}(50)x_C{}^2 - \tfrac{1}{2}(0.80)(1.2)^2 = -3.92x_C$$

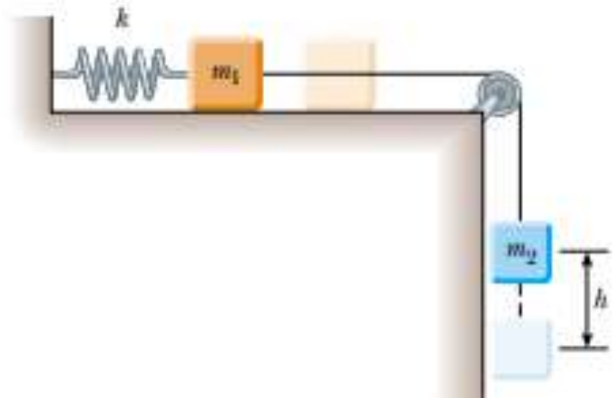
$$25x_C{}^2 + 3.92x_C - 0.576 = 0$$



Solving the quadratic equation for x_C gives $x_C = 0.092$ m and $x_C = -0.25$ m. The physically meaningful root is $x_C = 0.092$ m. The negative root does not apply to this situation because the block must be to the right of the origin

Example 8.10 Connected Blocks in Motion

Two blocks are connected by a light string that passes over a frictionless pulley, as shown in Figure 8.15. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.



(1) $$\Delta E_{mech} = \Delta U_g + \Delta U_s$$

(2) $$\Delta E_{mech} = -f_k h = -\mu_k m_1 g h$$

(3) $$\Delta U_g = U_{gf} - U_{gi} = 0 - m_2 g h$$

(4) $$\Delta U_s = U_{sf} - U_{si} = \tfrac{1}{2}kh^2 - 0$$

Substituting Equations (2), (3), and (4) into Equation (1) gives

$$-\mu_k m_1 g h = -m_2 g h + \tfrac{1}{2}kh^2$$

$$\mu_k = \frac{m_2 g - \tfrac{1}{2}kh}{m_1 g}$$

8.5 Relationship Between Conservative Forces and Potential Energy

The work done by a cons. force F as a particle moves along the *x axis is:*

$$W_c = \int_{x_i}^{x_f} F_x \, dx = -\triangle U$$

$$Or \triangle U = U_f - U_i = -\int_{x_i}^{x_f} F_x \, dx$$

Therefore, ΔU is negative when F_x and dx are in the same direction, as when an object is lowered in a gravitational field or when a spring pushes an object toward equilibrium.

We can then define the potential energy function as:

$$U_f(x) = -\int_{x_i}^{x_f} F_x \, dx + U_i$$

8.5 Relationship Between Conservative Forces and Potential Energy

If the point of application of the force undergoes an infinitesimal displacement d_x, *we can express the infinitesimal change in the potential energy of the system dU as*

$$dU = -F_x\, dx$$

Therefore, the conservative force is related to the potential energy function through the relationship

$$F_x = -\frac{dU}{dx}$$

That is, the x component of a conservative force acting on an object within a system equals the negative derivative of the potential energy of the system with respect to x.

Lecture Summary

If a particle of mass *m is at a distance y above the Earth's surface, the gravitational potential* energy of the particle–Earth system is

$$U_g = mgy$$

The elastic potential energy stored in a spring of force constant *k is*

$$U_s = \frac{1}{2}kx^2$$

Total Energy of A system is:

$$K_f + U_f = K_i + U_i$$

Lecture Summary

- A force is conservative if the work it does on a particle moving between two points is independent of the path the particle takes between the two points, Or if the work it does on a particle is zero when the particle moves through an arbitrary closed path and returns to its initial position. A force that does not meet these criteria is said to be nonconservative.
- The total mechanical energy of a system is defined as the sum of the kinetic energy and the potential energy:

$$E_{mech} = K + U$$

- If a system is isolated and if no nonconservative forces are acting on objects inside the system, then the total mechanical energy of the system is constant:

$$K_f + U_f = K_i + U_i$$

PROBLEMS

Section 8.1 Potential Energy of a System

2. A 400-N child is in a swing that is attached to ropes 2.00 m long. Find the gravitational potential energy of the child–Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.

SOLUTIONS TO PROBLEM:

$$U_g = mgy$$

$$y = 2$$

$$y = 2(1 - \cos\boldsymbol{\theta})$$

$$y = 0$$

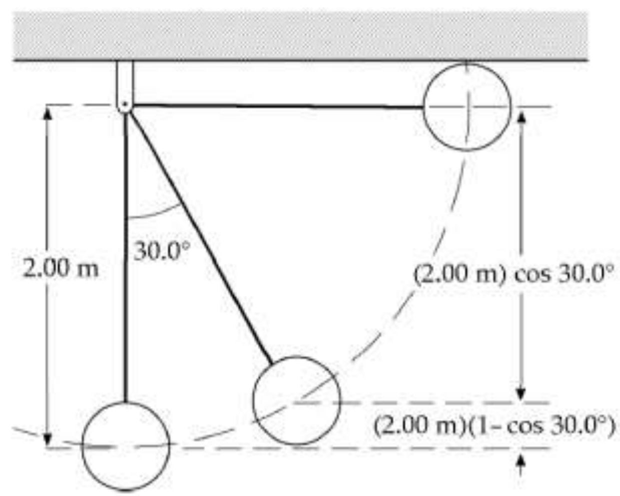


FIG. P8.2

PROBLEMS

Section 8.1 Potential Energy of a System

5. A bead slides without friction around a loop-the-loop (Fig. P8.5). The bead is released from a height *h* " 3.50*R*.

(a) What is its speed at point A?

(b) How large is the normal force on it if its mass is 5.00 g?

SOLUTIONS TO PROBLEM:

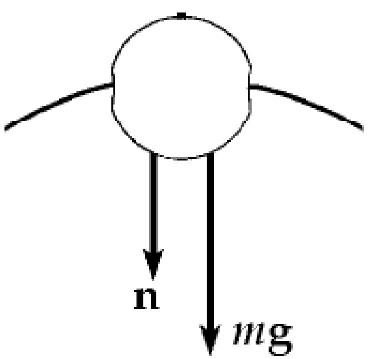


FIG. P8.5

$$K_i + U_i = K_f + U_f$$

$$mgh + 0 = mg(2R) + \frac{1}{2}mv^2$$

$$\sum F = m\frac{v^2}{R} = n + mg$$

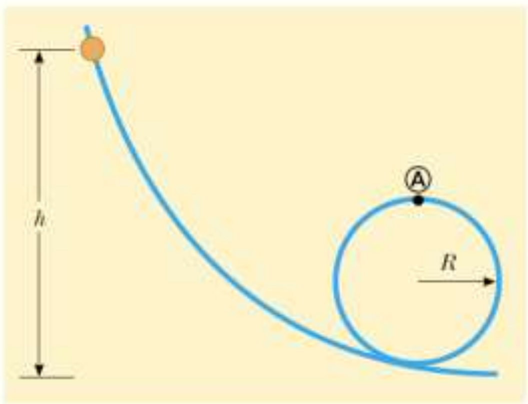


Figure P8.5

PROBLEMS

Section 8.1 Potential Energy of a System

6. Dave Johnson, the bronze medalist at the 1992 Olympic decathlon in Barcelona, leaves the ground at the high jump with vertical velocity component 6.00 m/s. How far does his center of mass move up as he makes the jump?

SOLUTIONS TO PROBLEM:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = 0 + mgy$$

PROBLEMS

Section 8.1 Potential Energy of a System

11. A block of mass 0.250 kg is placed on top of a light vertical spring of force constant 5 000 N/m and pushed downward so that the spring is compressed by 0.100 m. After the block is released from rest, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise?

SOLUTIONS TO PROBLEM:

From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si},$$

or

$$(0.250 \text{ kg})(9.80 \text{ m/s}^2)h = \left(\frac{1}{2}\right)(5\,000 \text{ N/m})(0.100 \text{ m})^2$$

This gives a maximum height $h = \boxed{10.2 \text{ m}}$.

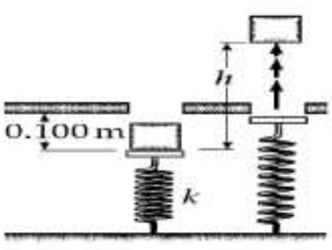


FIG. P8.11

PROBLEMS

Section 8.1 Potential Energy of a System

13. Two objects are connected by a light string passing over a light frictionless pulley as shown in Figure P8.13. The object of mass 5.00 kg is released from rest. Using the principle of conservation of energy, (a) determine the speed of the 3.00-kg object just as the 5.00-kg object hits the ground. (b) Find the maximum height to which the 3.00-kg object rises.

SOLUTIONS TO PROBLEM:

Using conservation of energy for the system of the Earth and the two objects

(a) $$(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00+3.00)v^2$$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$

$$\Delta y = 1.00\text{ m}$$

$$y_{max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

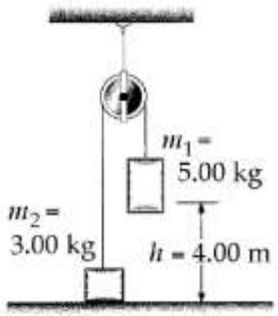


FIG. P8.13

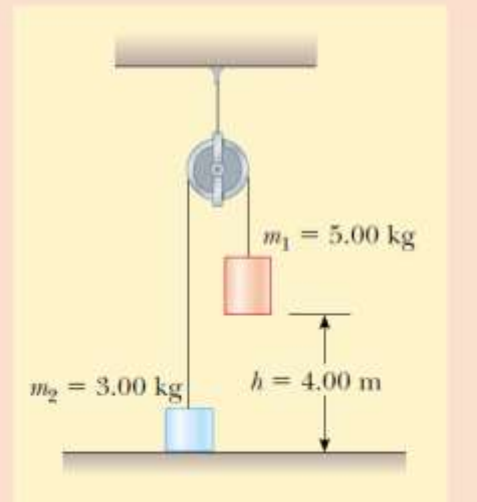


Figure P8.13 Problems 13 and 14.

PROBLEMS

Section 8.1 Potential Energy of a System

17. A 20.0-kg cannon ball is fired from a cannon with muzzle speed of 1 000 m/s at an angle of 37.0° with the horizontal. A second ball is fired at an angle of 90.0°. Use the conservation of energy principle to find (a) the maximum height reached by each ball and (b) the total mechanical energy at the maximum height for each ball. Let *y* =0 at the cannon.

SOLUTIONS TO PROBLEM:

(a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000 \sin 37.0°)^2}{2(9.80)} = \boxed{1.85 \times 10^4 \text{ m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10 \times 10^4 \text{ m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0 \text{ kg})(1\,000 \text{ m/s})^2 = \boxed{1.00 \times 10^7 \text{ J}}.$$

PROBLEMS

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

31. The coefficient of friction between the 3.00-kg block and the surface in Figure P8.31 is 0.400. The system starts from rest. What is the speed of the 5.00-kg ball when it has fallen 1.50 m?

SOLUTIONS TO PROBLEM:

$U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$: $m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}} = \boxed{3.74\ \text{m/s}}$$

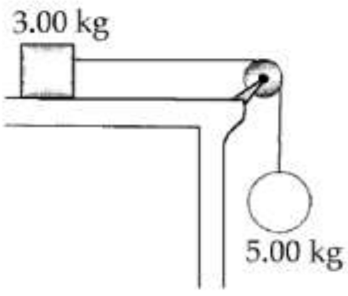


FIG. P8.31

PROBLEMS

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

33. A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s (Fig. P8.33). The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine

(a) the change in the block's kinetic energy, (b) the change in the potential energy of the block–Earth system, and (c) the frict block (assumed to be constant). (d) What is the cc

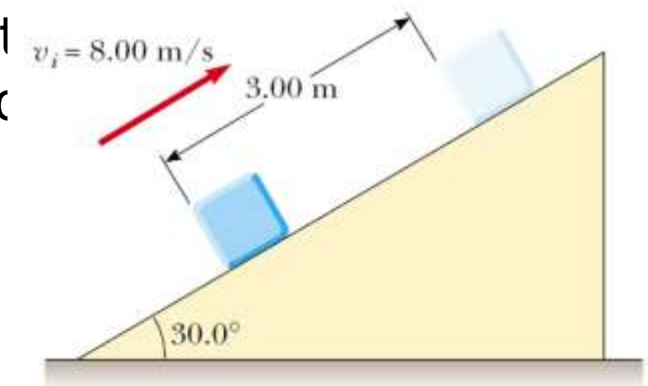


Figure P8.33

SOLUTIONS TO PROBLEM:

(a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\ \text{J}}$

(b) $\Delta U = mg(3.00\ \text{m})\sin 30.0° = \boxed{73.5\ \text{J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\ \text{J}}{3.00\ \text{m}} = \boxed{28.8\ \text{N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\ \text{N}$

$$\mu_k = \frac{28.8\ \text{N}}{(5.00\ \text{kg})(9.80\ \text{m/s}^2)\cos 30.0°} = \boxed{0.679}$$

PROBLEMS

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

36. A 50.0-kg block and a 100-kg block are connected by a string as in Figure P8.36. The pulley is frictionless and of negligible mass. The coefficient of kinetic friction between the 50.0 kg block and incline is 0.250. Determine the change in the kinetic energy of the 50.0-kg block as it moves from ! to ", a distance of 20.0 m.

SOLUTIONS TO PROBLEM:

$$\sum F_y = n - mg\cos 37.0° = 0$$

$$\therefore n = mg\cos 37.0° = 400\text{ N}$$

$$f = \mu n = 0.250(400\text{ N}) = 100\text{ N}$$

$$-f\Delta x = \Delta E_{\text{mech}}$$

$$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$$

$$\Delta U_A = m_A g(h_f - h_i) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times 10^3$$

$$\Delta U_B = m_B g(h_f - h_i) = (100)(9.80)(-20.0) = -1.96\times 10^4$$

$$\Delta K_A = \frac{1}{2}m_A\left(v_f^2 - v_i^2\right)$$

$$\Delta K_B = \frac{1}{2}m_B\left(v_f^2 - v_i^2\right) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$$

Adding and solving, $\Delta K_A = \boxed{3.92\text{ kJ}}$.

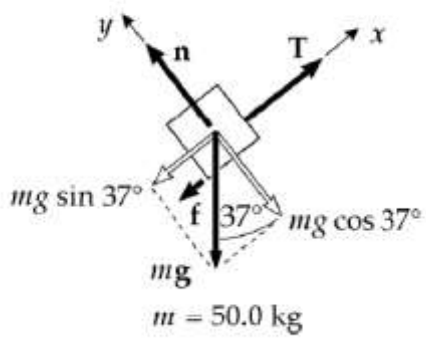


FIG. P8.36

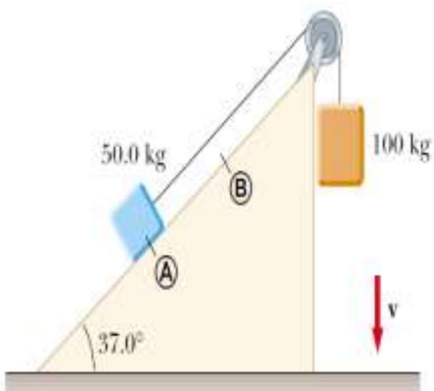


Figure P8.36

PROBLEMS

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

38. A 75.0-kg skysurfer is falling straight down with terminal speed 60.0 m/s. Determine the rate at which the skysurfer–Earth system is losing mechanical energy.

SOLUTIONS TO PROBLEM:

The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\ \text{kg})(9.80\ \text{m/s}^2)(60.0\ \text{m/s}) = \boxed{44.1\ \text{kW}}.$$

PROBLEMS

Section 8.5 Relationship Between Conservative Forces and Potential Energy

42. A potential-energy function for a two-dimensional force is of the form
$U = 3x^3y - 7x$.
Find the force that acts at the point (*x*, *y*).

SOLUTIONS TO PROBLEM:

$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial x} = -\left(9x^2y - 7\right) = 7 - 9x^2y$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y - 7x\right)}{\partial y} = -\left(3x^3 - 0\right) = -3x^3$$

Thus, the force acting at the point $(x,\ y)$ is $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7 - 9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

PROBLEMS

Additional Problems

55. Review problem. Suppose the incline is frictionless for the system described in Problem 54 (Fig. P8.54). The block is released from rest with the spring initially unstretched.

(a) How far does it move down the incline before coming to rest?

(b) (b) What is its acceleration at its lowest point? Is the acceleration constant?

(c) Describe the energy transformations that occur during the descent.

SOLUTIONS TO PROBLEM:

(a) Since no nonconservative work is done, $\Delta E = 0$

Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg\sin\theta)x$

and $U_f = \frac{1}{2}kx^2$

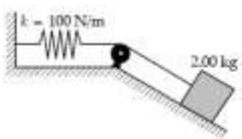


FIG. P8.55

Substituting values yields $(2.00)(9.80)\sin 37.0° = (100)\frac{x}{2}$ and solving we find

$$x = \boxed{0.236\text{ m}}$$

(b) $\sum F = ma$. Only gravity and the spring force act on the block, so

$$-kx + mg\sin\theta = ma$$

For $x = 0.236$ m,

$a = \boxed{-5.90\text{ m/s}^2}$. The negative sign indicates a is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) U(gravity) decreases monotonically as the height decreases.
U(spring) increases monotonically as the spring is stretched.
K initially increases, but then goes back to zero.

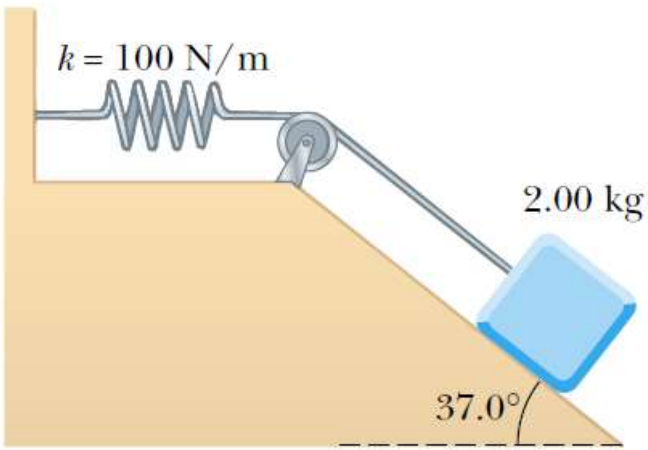


Figure P8.54 Problems 54 and 55.

PROBLEMS

Additional Problems

57. A 10.0-kg block is released from point A in Figure P8.57. The track is frictionless except for the portion between points B and C , which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2 250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily.

Determine the coefficient of kinetic friction between the block and the rough surface between B and C.

SOLUTIONS TO PROBLEM:

$$\Delta E_{\text{mech}} = -f\Delta x$$

$$E_f - E_i = -f \cdot d_{BC}$$

$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

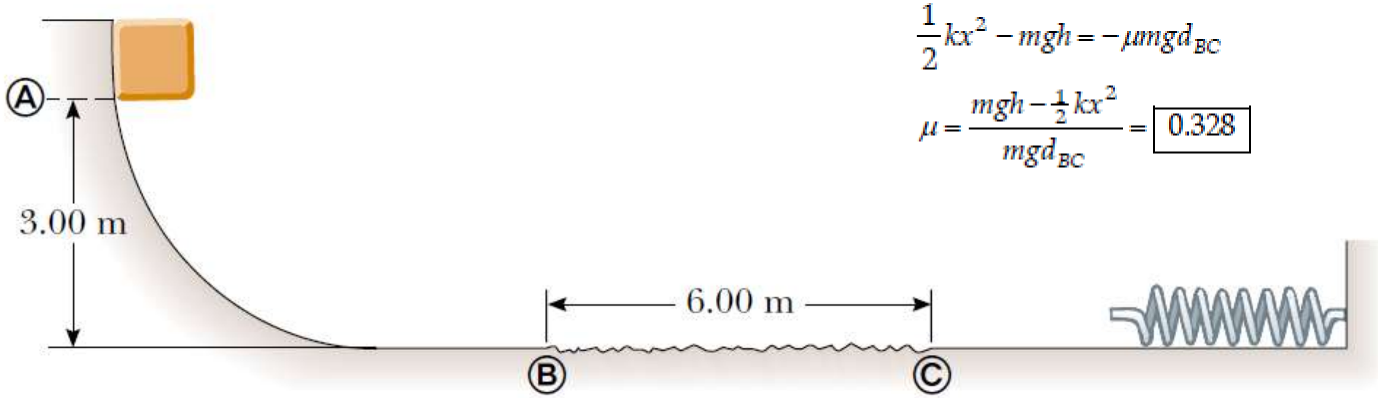


Figure P8.57

PROBLEMS

Additional Problems

59. A 20.0-kg block is connected to a 30.0-kg block by a string that passes over a light frictionless pulley. The 30.0-kg block is connected to a spring that has negligible mass and a force constant of 250 N/m, as shown in Figure P8.59. The spring is unstretched when the system is as shown in the figure, and the incline is frictionless. The 20.0-kg block is pulled 20.0 cm down the incline (so that the 30.0-kg block is 40.0 cm above the floor) and released from rest.

Find the speed of each block when the 30.0-kg block is 20.0 cm above the floor (that is, when the spring is unstretched).

SOLUTIONS TO PROBLEM:

$$(K+U)_i = (K+U)_f$$

$$0 + (30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) + \frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

$$= \frac{1}{2}(50.0\text{ kg})v^2 + (20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J} + 5.00\text{ J} = (25.0\text{ kg})v^2 + 25.2\text{ J}$$

$$\boxed{v = 1.24\text{ m/s}}$$

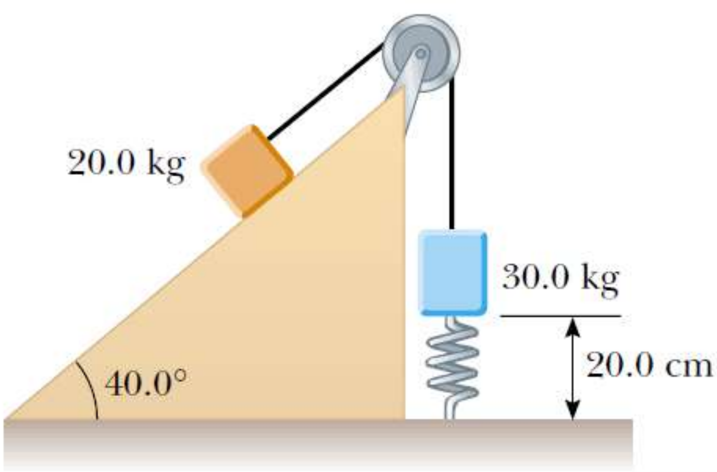


Figure P8.59

PROBLEMS

Additional Problems

60. A 1.00-kg object slides to the right on a surface having a coefficient of kinetic friction 0.250 (Fig. P8.60). The object has a speed of *vi* " 3.00 m/s when it makes contact with a light spring that has a force constant of 50.0 N/m. The object comes to rest after the spring has been compressed a distance *d*. The object is then forced toward the left by the spring and continues to move in that direction beyond the spring's unstretched position. Finally, the object comes to rest a distance *D* to the left of the unstretched spring. Find (a) the distance of compression *d*, (b) the speed *v* at the unstretched position when the object is moving to the left, and (c) the distance *D* where the object comes to rest.

SOLUTIONS TO PROBLEM:

(a) Between the second and the third picture, $\Delta E_{mech} = \Delta K + \Delta U$

$$-\mu m g d = -\frac{1}{2} m v_i^2 + \frac{1}{2} k d^2$$

$$\frac{1}{2}(50.0\ \text{N/m})d^2 + 0.250(1.00\ \text{kg})(9.80\ \text{m/s}^2)d - \frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s}^2) = 0$$

$$d = \frac{[-2.45 \pm 21.25]\ \text{N}}{50.0\ \text{N/m}} = \boxed{0.378\ \text{m}}$$

(b) Between picture two and picture four, $\Delta E_{mech} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2} m v^2 - \frac{1}{2} m v_i^2$$

$$v = \sqrt{(3.00\ \text{m/s})^2 - \frac{2}{(1.00\ \text{kg})}(2.45\ \text{N})(2)(0.378\ \text{m})}$$

$$= \boxed{2.30\ \text{m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{mech} = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s})^2$$

$$D = \frac{9.00\ \text{J}}{2(0.250)(1.00\ \text{kg})(9.80\ \text{m/s}^2)} - 2(0.378\ \text{m}) = \boxed{1.08\ \text{m}}$$

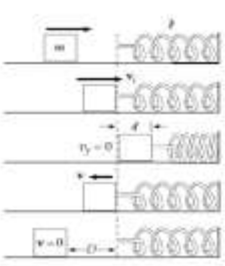


FIG. P8.60