EE:211, Computational Techniques in Electrical Engineering

Mid#2 Exam (2^{nd} Semester, Spring 2012,1432/33H) *Name:* ______________________

Wednesday, April 25, 2012

Total Marks= 40 Student-ID: ______________________

Total Time= 180 Minutes

Problem#1[8] Find the function $P(x) = a + bcos(\pi x) + csin(\pi x)$, which interpolates the data:

x	1	1.5	2
y	2	5	4

Problem#2[8] The following data are taken from a polynomial p(x) of degree ≤ 5. What is the polynomial and what is the degree. Simplify your result as much as possible.

x	-2	-1	0	1	2	3
y	-5	1	1	1	7	25

Problem#3[8] The length of the curve represented by a function y = f(x) on an interval [a , b] is given by the following integral:

$$I(f) = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

Using the Trapezoidal rule and n = 4, compute the length of the curve when

$$f(x) = e^{\cos(x)}, \quad 0 \le x \le \frac{\pi}{4}$$

Problem#4[8] Let $I_h(f) = \frac{3h}{4}[f(0) + 3f(2h)]$

What is the degree of the precision of the approximation: $I_h(f) \approx \int_0^{3h} f(x)dx$

Problem#5[8] Solve the linear system using partial pivoting:

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & 5 & 0 \\ -4 & 1 & 2 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

EE : 211, Solution, Mid # 2

Spring : 2012

Q #1 $P(x) = a + b\cos(\pi x) + c\sin(\pi x)$

$P(1) = a - b = 2 \longrightarrow (1)$

$P(1.5) = a - c = 5 \longrightarrow (2)$

$P(2) = a + b = 4 \longrightarrow (3)$

From eq (1) and eq (3) $a = 3$

$b = 1$

From eq (2) $c = -2$

$P(x) = 3 + \cos(\pi x) - 2\sin(\pi x)$

Q # 2 Construct Newton's Polynomial by using Divided Difference table.

i	x_i	$f(x_i)$	$Df(x_i)$	$D^2f(x_i)$	$D^3f(x_i)$	$D^4f(x_i)$	D^5
0	-2	-5	6	-3	1	0	0
1	-1	1	0	0	1	0	X
2	0	1	0	3	1	X	X
3	1	1	6	6	X	X	X
4	2	7	18	X	X	X	X
5	3	25	X	X	X	X	X

use $f[x_0, x_1] = 6$ $\quad f[x_0, x_1, x_2] = -3$

$f[x_0, x_1, x_2, x_3] = 1$ $\quad$ Higher divided difference $= 0$

$$P_1(x) = -5 + 6(x+2)$$

$$P_2(x) = P_1(x) - 3(x+2)(x+1)$$

$$P_3(x) = P_2(x) + 1(x+2)(x+1)(x)$$

$$P_4(4) = P_3(x) + 0(x+2)(x+1)(x)(x-1) = P_3(x)$$

Degree of Polynomial (Highest) $= 3$

$$P_3(x) = -5 + 6(x+2) - 3(x+2)(x+1) + x(x+1)(x+2)$$

$$= -5 + 6x + 12 - 3x^2 - 6 - 9x + x^3 + 3x^2 + 2x$$

$$\boxed{P_3(x) = x^3 - x + 1}$$

$\boxed{Q\#2}$ Solution by Lagrange's Method

$$L_0(x) = \frac{(x+1)(x)(x-1)(x-2)(x-3)}{(-1)(-2)(-3)(-4)(-5)}$$

$$L_0(x) = -0.0083x^5 + 0.0417x^4 - 0.0417x^3 - 0.0417x^2 + 0.05x$$

$$L_1(x) = 0.0417x^5 - 0.1667x^4 - 0.0417x^3 + 0.667x^2 - 0.5x$$

$$L_2(x) = -0.0833x^5 + 0.25x^4 + 0.4167x^3 - 1.25x^2 - 0.3333x + 1$$

$$L_3(x) = 0.0833x^5 - 0.1667x^4 - 0.5833x^3 + 0.6667x^2 + x$$

$$L_4(x) = -0.0417x^5 + 0.0417x^4 + 0.2917x^3 - 0.0417x^2 - 0.25x$$

$$L_5(x) = 0.0083x^5 - 0.0417x^3 + 0.0333x$$

$$P_5(x) = y_0L_0 + y_1L_1 + y_2L_2 + y_3L_3 + y_4L_4 + y_5L_5$$

$$P_3(x) = x^3 - x + 1$$

Final degree : 3

Q #3 $\quad I(f) = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$f(x) = e^{\cos(x)} \qquad f'(x) = -\sin(x)\, e^{\cos(x)}$

$$I(f) = \int_a^b \sqrt{1 + \sin^2(x)\, e^{2\cos(x)}}\, dx.$$

$a = 0 \qquad b = \pi/4.$

Trapezoidal Rule: $\left[\begin{array}{l} x_j = a + jh \\ j = 0, 1, \dots, n \end{array} \right.$ $\qquad a = 0 \quad h = \frac{b-a}{n} = \frac{\pi}{16}$

$n = 4$

$x_0 = 0 \qquad x_1 = \frac{\pi}{16} \qquad x_2 = \frac{\pi}{8} \qquad x_3 = \frac{3\pi}{16} \qquad x_4 = \frac{\pi}{4}$

let $\quad g(x) = \sqrt{1 + \sin^2(x)\, e^{2\cos(x)}}$

$x_0 : \quad g(x_0) = g(0) = \sqrt{1} = 1$

$x_1 : \quad g(x_1) = \sqrt{1 + \sin^2\left(\frac{\pi}{16}\right) e^{2\cos\left(\frac{\pi}{16}\right)}} = 1.12722$

$x_2 = \quad g(x_2) = \sqrt{1 + \sin^2\left(\frac{\pi}{8}\right) e^{2\cos\left(\frac{\pi}{8}\right)}} = 1.389$

$x_3 : \quad g(x_3) = \sqrt{1 + \sin^2\left(\frac{3\pi}{16}\right) e^{2\cos\left(\frac{3\pi}{16}\right)}} = 1.6211$

$x_4 : \quad g(x_4) = 1.7483$

$$T_4(f) = h\left[\frac{g(x_0)}{2} + g(x_1) + g(x_2) + g(x_3) + \frac{g(x_4)}{2} \right]$$

$$= \frac{\pi}{16}\left[\frac{1}{2} + 1.12722 + 1.389 + 1.6211 + \frac{1.7483}{2} \right]$$

$$\boxed{T_4(f) = 1.0822}$$

Q #4 Use $f(x) = 1, x, x^2, x^3, x^4, \ldots$

$$\frac{3h}{4}\left[f(0) + 3f(2h)\right] = \int_0^{3h} f(x)\,dx.$$

$\boxed{f(x) = 1}$ $\quad \frac{3h}{4}\left[1 + 3\right] = \int_0^{3h} (1)\,dx.$

$$\boxed{3h = 3h}$$

$\boxed{f(x) = x}$ $\quad \frac{3h}{4}\left[0 + 3 \times 2h\right] = \int_0^{3h} x\,dx = \frac{1}{2}(3h)^2$

$$\boxed{\frac{9h^2}{2} = \frac{9h^2}{2}}$$

$\boxed{f(x) = x^2}$ $\quad \frac{3h}{4}\left[0 + 3 \times (2h)^2\right] = \int_0^{3h} x^2\,dx = \frac{1}{3}(3h)^3$

$$\boxed{9h^3 = 9h^3}$$

$\boxed{f(x) = x^3}$ $\quad \frac{3h}{4}\left[0 + 3(2h)^3\right] = \int_0^{3h} x^3\,dx = \frac{1}{4}(3h)^4$

$$\boxed{18h^4 \neq \frac{81}{4}h^4}$$

Hence Error $\neq 0$ for $f(x) = x^3$

$$\boxed{\text{Degree of Precision} = 2}$$

Q. #5 **Linear System Solution with Partial Pivoting**

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 1 \\ 2 & 3 & 5 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 \\ -4 & 1 & 2 & 3 & 0 \end{array}\right]$$

$R_1 \longleftrightarrow R_4$

$$\left[\begin{array}{cccc|c} -4 & 1 & 2 & 3 & 0 \\ 2 & 3 & 5 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 \\ 1 & 2 & 3 & 1 & 1 \end{array}\right]$$

$m_{21} = -\frac{1}{2} \qquad m_{31} = -\frac{3}{4} \qquad m_{41} = -\frac{1}{4}$

$$\left[\begin{array}{cccc|c} -4 & 1 & 2 & 3 & 0 \\ 0 & 3.5 & 6 & 1.5 & 0 \\ 0 & 4.75 & 6.5 & 2.25 & 1 \\ 0 & 2.25 & 3.5 & 1.75 & 1 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$

$$\left[\begin{array}{cccc|c} -4 & 1 & 2 & 3 & 0 \\ 0 & 4.75 & 6.5 & 2.25 & 1 \\ 0 & 3.5 & 6 & 1.5 & 0 \\ 0 & 2.25 & 3.5 & 1.75 & 1 \end{array}\right]$$

$m_{32} = 0.7368 \qquad m_{42} = 0.4737$

$$\left[\begin{array}{cccc|c} -4 & 1 & 2 & 3 & 0 \\ 0 & 4.75 & 6.5 & 2.25 & 1 \\ 0 & 0 & 1.2108 & -0.1578 & -0.7368 \\ 0 & 0 & 0.421 & 0.6842 & 0.5263 \end{array}\right]$$

$m_{43} = 0.3477$

$$\left[\begin{array}{cccc|c} -4 & 1 & 2 & 3 & 0 \\ 0 & 4.75 & 6.5 & 2.25 & 1 \\ 0 & 0 & 1.2108 & -0.1578 & -0.7368 \\ 0 & 0 & 0 & 0.7391 & 0.7825 \end{array}\right]$$

$x_4 = 1.059$

$x_3 = -0.4705$

$x_2 = 0.3527$

$x_1 = 0.6472$