King Saud University Department of Mathematics

Mid Term Exam 280-Math 1Semester (1444)

Question 1 [4] Let A and B be non-empty bounded sets of positive real numbers such that Inf(B)>0. Define the set $\frac{A}{B} = \left\{\frac{a}{b}: \quad a \in A,\ b \in B\right\}$.

Show that Sup$(\frac{A}{B}) = \frac{Sup(A)}{Inf(B)}$.

Question 2 [3]

Let $A = \{\ \sqrt{n+1} - \sqrt{n},\quad n\ \in \mathbb{N}\}$. Determine sup A and Inf A where they exist.

Question 3 [3] Use the definition of convergence to prove that

$$\lim_{n\to\infty} \frac{10n^2}{n^2+16n+1} = 10.$$

Question 4 [4]

Determine whether the sequence $(\frac{\mathrm{n}-\cos(n)}{n})$ is convergent or divergent, and find the limit where it exists.

Question 5 [4]

Prove that $\{\frac{n^2-1}{n^2}\}$ is Cauchy using directly the definition of Cauchy sequences.

Question 6 [4+4+4]

Test the following series for convergence:

a. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

c. $\sum_{n=1}^{\infty} e^{-n^2}$