**Chapter 6 – 105 STAT**

**Example 4: Page 17/31**



$$H\_{0}:The data fitted Normal\left(μ,σ^{2}\right)$$

$$H\_{1}:The data not fitted Normal(μ,σ^{2})$$

The value of Known parameters $μ,σ^{2}$ are unknown ( m=2)

**First find :** $ μ,σ^{2}$$$\overbar{X}=\sum\_{i=1}^{7}\frac{O\_{i}\* x\_{i}}{n}=$$

$$\frac{5\*45+16\*50+28\*55+32\*60+23\*65+11\*70+5\*75}{120}=59.4$$

$$S^{2}=\sum\_{i=1}^{7}\frac{O\_{i}\*\left(x\_{i}-\overbar{X}\right)}{n-1}=$$

$$=\frac{5\*\left(45-59.4\right)+16\*\left(50-59.4\right)+28\*\left(55-59.4\right)+32\*\left(60-59.4\right)+}{120-1}$$

$$\frac{+23\*(65-59.4)+11\*(70-59.4)+5\*(75-59.4)}{120-1}=51.9$$

**Then,**

**Hypothesis are**

$$H\_{0}:The data fitted Normal\left(59.4, 51.9\right)$$

$$H\_{1}:The data not fitted Normal(59.4, 51.9)$$

**Second :**

The class width = Midclass2 – Midclass1 = 50 - 45 = 5 , $\frac{width}{2}=\frac{5}{2}=2.5$

**To find class interval ::** Lower limit = Midclass - 2.5

 upper limit = Midclass + 2.5

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Class | Mid class | (Mid class-2.5,Mid class+2.5) | Class boundaries | Frequency$$O\_{i}$$ |  |  |
| 1 | 45 | 45-2.5 **–** 45 +2.5 | 42.5 – 47.5 | 5 |  |  |
| 2 | 50 | 50-2.5 **–** 55 +2.5 | 47.5 – 52.5 | 16 |  |  |
| 3 | 55 | 55-2.5 **–** 60 +2.5 | 52.5 – 57.5 | 28 |  |  |
| 4 | 60 | 60-2.5 **–** 65 +2.5 | 57.5 – 62.5 | 32 |  |  |
| 5 | 65 | 65-2.5 **–** 70 +2.5 | 62.5 – 67.5 | 23 |  |  |
| 6 | 70 | 70-2.5 **–** 70 +2.5 | 67.5 – 72.5 | 11 |  |  |
| 7 | 75 | 75-2.5 **–** 75 +2.5 | 72.5 – 77.5 | 5 |  |  |
|  |  |  |  |  |  |  |

Find the values of Z$$Z=\frac{X\_{i}-3.5}{\sqrt{51.9}}=\frac{X\_{i}-3.5}{0.7}$$

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Class | Class boundaries | Frequency$$O\_{i}$$ | $$Z=\frac{X\_{i}-59.4}{7.20}$$ |  Z |
| 1 | 42.5 – 47.5 | 5 | $Z\_{1}=\frac{42.5-59.4}{7.20}$ = -2.35 | -2.35 |
| 2 | 47.5 – 52.5 | 16 | $Z\_{2}=\frac{47.5-59.4}{7.20}$ = -1.65 | -1.65 |
| 3 | 52.5 – 57.5 | 28 | $Z\_{3}=\frac{52.5-59.4}{7.20}$ = - 0.96 | -0.96 |
| 4 | 57.5 – 62.5 | 32 | $Z\_{4}=\frac{57.5-59.4}{7.20}$= -0.26 | -0.26 |
| 5 | 62.5 – 67.5 | 23 | $Z\_{5}=\frac{62.5-59.4}{7.20}$ = 0.43 | 0.43 |
| 6 | 67.5 – 72.5 | 11 | $Z\_{6}=\frac{67.5-59.4}{7.20}$ = 1.13 | 1.13 |
| 7 | 72.5 – 77.5 | 5 | $Z\_{7}=\frac{72.5-59.4}{7.20}$ = 1.82 | 1.82 |
|  |  |  | $Z\_{8}=\frac{77.5-59.4}{7.20}$ = 2.51 | 2.51 |

Find the value of $P\_{i}=P\left(X\_{i}<X<X\_{i+1}\right)=From Z-table$

$$Z\_{i}=\frac{X\_{i}- μ}{σ}= \frac{X\_{i}-59.4}{7.20}$$

 $P\_{1}=P\left(42.5<X<47.5\right)=$

 $P\left(-2.35<Z<-1.65\right)$=$0.0495-0.0094=0.0383$

$$P\_{2}=P\left(47.5<X<52.5\right)=$$

$P\left(-1.65<Z<-0.96\right)$=$0.1685-0.0495=0.119$

$$P\_{3}=P\left(52.5<X<57.5\right)=$$

 $P\left(-0.96<Z<-0.26\right)$=$0.3947-0.1685=0.2262$

$$P\_{4}=\left(57.5<X<62.5\right)=$$

 $P\left(-0.26<Z<0.43\right)$=$0.6664-0.3947=0.2717$

$$P\_{5}=\left(62.5<X<67.5\right)=$$

 $P\left(0.43<Z<1.13\right)$=$0.8708-0.6664=0.2044$

$$P\_{6}=\left(67.5<X<72.5\right)=$$

 $P\left(1.13<Z<1.82\right) $=$0.9656-0.8708=0.0948$

$$P\_{7}=\left(72.5<X<77.5\right)=$$

 $P\left(1.82<Z<2.51\right)$=$0.9940-0.9656=0.0284$

**Then ,value of** $E\_{i}=n x P\_{i}=120x P\_{i}$

$E\_{1}=120x P\_{1}=120x0.0383=$ 4.60 $(E\_{1}<5)$

$E\_{2}=120x P\_{2}=120x0.119=$14.28

$$E\_{3}=120x P\_{3}=120x0.2262=27.14$$

$$E\_{4}=120x P\_{4}=120x0.2717=32.60$$

$$E\_{5}=120x P\_{5}=120x0.2044=24.53$$

$$E\_{6}=120x P\_{6}=120x0.0948=11.38$$

$E\_{7}=120x P\_{7}=120x0.0284=3.41$ $(E\_{7}<5)$

The table is

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Class | 1 | 2 | 3 | 4 | 5 |
| Class boundaries | 42.5-47.547.5-52.5 | 52.5-57.5 | 57.5-62.5 | 62.5-67.5 | 67.5-72.572.5-77.5 |
| Observed(O) | 5+16 | 28 | 32 | 23 | 11+5 |
| Expected (E) | 4.60+14.28 | 27.14 | 32.60 | 24.53 | 11.38+3.41 |

 **Combine intervals and Add Observed and Expected, the table now**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Class | 1 | 2 | 3 | 4 | 5 |
| Class boundaries | 42.5-52.5 | 52.5-57.5 | 57.5-62.5 | 62.5-67.5 | 67.5-77.5 |
| Observed(O) | 21 | 28 | 32 | 23 | 16 |
| Expected (E) | 18.88 | 27.14 | 32.60 | 24.53 | 14.79 |
| $$o\_{i}-E\_{i}$$ | 2.12 | 0.86 | -0.6 | -1.53 | 1.21 |

**The test is** $ χ^{2}=\sum\_{i=1}^{4}\frac{(O\_{i}-E\_{i})^{2}}{E\_{i}}$

=$\frac{(2.12)^{2}}{18.88}+\frac{(0.86)^{2}}{27.14}+\frac{(-0.6)^{2}}{32.60}+\frac{(-1.53)^{2}}{24.53}++\frac{(1.21)^{2}}{14.79}$= 0.47077

**To find the** $ χ^{2}\_{α}$ : where α =0.05

 $df=v=k-m-1=4-2-1=1$

**Reject H0 if** $ χ^{2}> χ^{2}\_{α}$

 2.54 > 3. 481 (not satisfy)

Accept H0 : The data is Normal N(59.4,51.9)

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