Exercises

liner programming problem

1.1) A diet conscious housewife wishes to ensure certain minimum intake of vitamins A, B and C for the family. The minimum daily (quantity) needs of vitamins A,B and C for the family are respectively 30, 20 and 16 for the supply of theses minimum vitamin requirements, the house wife relies on two fresh foods. The **first** one provides 7, 5, 2 units of the three vitamins per gram respectively and the **second** one provides 2, 4, 8 units of the same three vitamins per gram of the foodstuff respectively. The first foodstuff costs 3$ per gram and the second 2$ per gram. **The problem is how many grams of each foodstuff should the housewife buy every day to keep her food bills as low as possible?** (Formulate the problem as liner programming problem.)

Answer:

let x_1: *the number of* units of food 1.

let x_2: *the number of* units of foods 2.

The data of the given problem can be summarized as below:

Food	Content of vitamins type			Cost per unit ($)
	A	B	C	
x_1	7	5	2	3
x_2	2	4	8	2
Minimum vitamins required	30	20	16	

Objective function:

Minimum $Z = 3x_1 + 2x_2$

subject to the Constraints:

$$(1)\quad 7x_1 + 2x_2 \geq 30$$

$$(2)\quad 5x_1 + 4x_2 \geq 20$$

$$(3)\quad 2x_1 + 8x_2 \geq 16$$

$$(4)\quad x_1, x_2 \geq 0$$

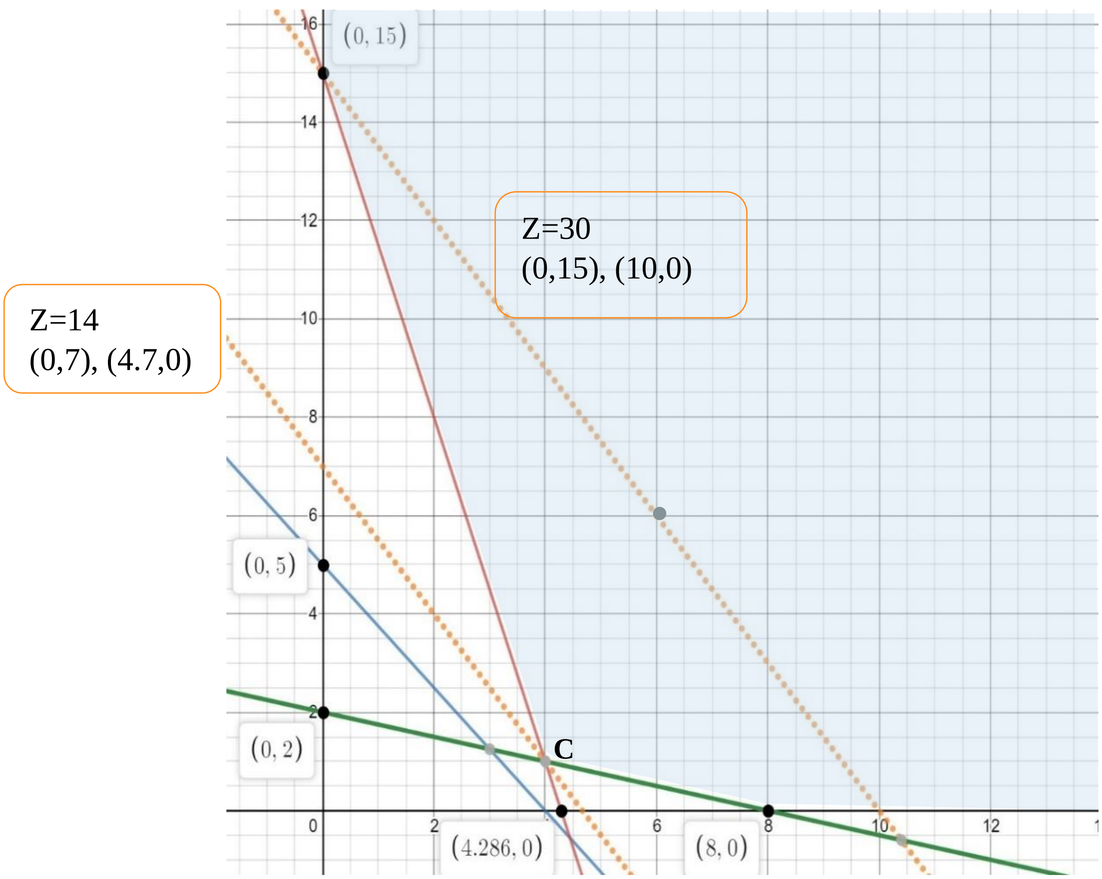
- Solve graphically a Linear Programming model that will allow the housewife to minimize the cost. And determine the optimal solution.

To determine two points on the Constraints as follow

$7x_1 + 2x_2 = 30$ >> (0 , 15) and (4.3 , 0)

$5x_1 + 4x_2 = 20$ >> (0 , 5) and (4, 0)

$2x_1 + 8x_2 = 16$ >> (0,2) and (8,0)



To determine the direction of solution region for each constraints:

Constraint 1

Point above line (4,8)

$7(4) + 2(8) \geq 30$

$44 \geq 30$

Point under line (2,4)

$7(2) + 2(4) \geq 30$

$26 \ngeq 30$

Constraint 2

(2,4)

$5(2) + 4(4) \geq 20$

$26 \geq 20$

(2,2)

$5(2) + 4(2) \geq 20$

$18 \ngeq 20$

Constraint 3

(2,2)

$2(2) + 8(2) \geq 16$

$20 \geq 16$

(1,1)

$2(1) + 8(1) \geq 16$

$10 \ngeq 16$

To plot objective function line, which pass through pint (6,6)

$3(6) + 2(6) = 30$

We need other point $X_1 = 0$

$2(X_2) = 30$

$X_2 = 15$

(6,6); (0,15) ; (10,0)

The optimal solution of an LPP occurs at point C. The values of associated with the optimum point C are determined by solving the equations associated with lines (1) and (3), that is,

$4^* \; (7x_1 + 2x_2 = 30 \;)$

$(-)^* \; (2x_1 + 8x_2 = 16 \;)$

$(28 - 2)x_1 = 120 - 16$

$26x_1 = 104 \ggg \; x_1{}^* = 4$

$7(4) + 2x_2 = 30$

$x_2{}^* = \frac{30-28}{2} = 1$

Or we can find the optimal solution by compute the value of the objective function at each vertex (Extreme) pints, as follows

Points (x_1, x_2)	$Z = 3x_1 + 2x_2$
(0,15)	30
(4,1)	**14**
(8,0)	24

We have unique optimal solution at $x_1^* = 4$, $x_2^* = 1$ with an optimal value Z=14 .

https://www.desmos.com/calculator/3f8hz2hvu4

1.2) The manager of an oil refinery has to decide the optimal production amount of the two processes with the following data in the table:

Process	input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	5	3	5	8
2	4	5	4	4

The maximum amount available of crude A and B are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The **profits** per production run from process 1 and process 2 are 3$ and 4$ respectively. **Formulate the problem as liner programming problem.**

Answer:

let x_1:the number of production runs from processes 1.

let x_2: the number of production runs from processes 2.

Objective function:

Maximize $Z = 3x_1 + 4x_2$

Constrains:

(1) $5x_1 + 4x_2 \leq 200$ Maximum amounts of crude A available

(2) $3x_1 + 5x_2 \leq 150$ Maximum amounts of crude B available

(3) $5x_1 + 4x_2 \geq 100$ Minimum amount of gasoline X to be produced

(4) $8x_1 + 4x_2 \geq 80$ Minimum amount of gasoline Y to be produced

(5) $x_1 , x_2 \geq 0$

1.3) A workshop has three (3) types of machines A, B and C; it can manufacture two (2) products 1 and 2, and all products have to go to each machine and each one goes in the same order; First to the machine A, then to B and then to C. The following table shows:

- The hours needed at each machine, per product unit.
- The total available hours for each machine, per week.
- The profit of each product per unit sold.

Type of Machine	Product 1	Product 2	Available hours per week
A	2	2	16
B	1	2	12
C	4	2	28
Profit per unit	1	1.50	

Formulate and solve using the **graphical method** a **Linear Programming model** for the previous situation that allows the workshop to obtain maximum gains.

Answer:

let x_1:the number of units of product 1 per week.

let x_2:the number of units of product 2 per week.

Objective function:

Maximize $Z = x_1 + 1.5x_2$

Constrains:

(1) $2x_1 + 2x_2 \leq 16$

(2) $x_1 + 2x_2 \leq 12$

(3) $4x_1 + 2x_2 \leq 28$

(4) $x_1 , x_2 \geq 0$

For the graphical solution:

Step 1: since $x_1 , x_2 \geq 0$, we consider only the first quadrant of x y-plane.

Step 2: we draw straight lines for the following equations:

$$2x_1 + 2x_2 = 16$$

$$x_1 + 2x_2 = 12$$

$$4x_1 + 2x_2 = 28$$

To determine two point on the straight line $2x_1 + 2x_2 = 16$
let $x_2 = 0$ >> $x_1 = 8$, (8,0) is a point on the line1.
let $x_1 = 0$ >> $x_2 = 8$, (0,8) is a point on the line1.
To determine two point on the straight line $x_1 + 2x_2 = 12$
let $x_2 = 0$ >> $x_1 = 12$, (12,0) is a point on the line 2.
let $x_1 = 0$ >> $x_2 = 6$, (0,6) is a point on the line 2.
To determine two point on the straight line $4x_1 + 2x_2 = 28$
let $x_2 = 0$ >> $x_1 = 14$, (7,0) is a point on the line 3.
let $x_1 = 0$ >> $x_2 = 14$, (0,14) is a point on the line 3.

https://www.desmos.com/calculator/cgc1f18x0f

The intersection of region is the feasible solution of LPP (linear programming problem). Therefore, every point in the shaded region is a feasible solution of LPP, since this point satisfies all the constraints including the non-negative constraints.

Technique to find the optimal solution of an LPP

Step 1: Find the coordinates of each vertex (corner point-Extreme point) of the feasible region. These coordinates can be obtained from the graph ***or*** by solving the equations of the lines.

Step 2: at each vertex compute the value of objective function.

Step 3: Identify the vertex at which the value of objective function is maximum.

Points (x_1, x_2)	$Z = x_1 + 1.5x_2$
(0,0)	0
(0,6)	0.25
(4,4)	**10**
(6,2)	9
(7,0)	7

The optimal solution at $x_1 = 4$, $x_2 = 4$ with an optimal value Z=10 that represents the workshop's profit.

#Coordinates can be obtained by solving the equations of the binding constraint, as following

$$2x_1 + 2x_2 = 16$$
(-)* $(x_1 + 2x_2 = 12\ )$

$$(2x_1 - x_1) + (2x_2 - 2x_2) = 16 - 12$$
$$x_1 = 4$$
$$2(4) + 2x_2 = 16$$
$$x_2 = \frac{16-8}{2} = 4$$
>> (4,4)

$$2x_1 + 2x_2 = 16$$
(-)* $(4x_1 + 2x_2 = 28\ )$

$$-2x_1 = -12$$
$$x_1 = 6$$
$$x_2 = \frac{16-12}{2} = 2$$
>> (6,2)

*Note: we can use the **Graphic Linear Optimizer (GLP)** software to solution like this model.

https://www.desmos.com/calculator/2rnqgoa6a4

HW (1.4) A company produces two different products. One of them needs 1/4 of an hour of assembly work per unit (عمل التجميع), 1/8 of an hour in quality control work (اعمال ضبط الجوده) and US$**1.2** in raw materials. The other product requires 1/3 of an hour of assembly work per unit, 1/3 of an hour in quality control work and US$**0.9** in raw materials. Given the current availability of staff in the company, each day there is at most a total of **90** hours available for **assembly** and **80** hours for **quality control**. The first and second products described have a market value (sale price) of US$**9.0** and $**8.0** per unit respectively. In addition, the maximum amount of daily sales for the first product is estimated to be 200 units, without there being a maximum limit of daily sales for the second product.

Formulate and solve graphically a Linear Programming model that will allow the company to maximize profits.