Exercise

Solve the following LPP using simplex method:

1- Max $Z = 3X_1 + 4X_2$

Subject to

$15X_1 + 10X_2 \le 300$

$2.5X_1 + 5X_2 \le 110$

$X_1 \ge 0, X_2 \ge 0$

Solution: (we have canonical form)

The standard form of LPP

Max $Z - 3X_1 - 4X_2 = 0$

Subject to

$15X_1 + 10X_2 + S_1 = 300$

$2.5X_1 + 5X_2 + S_2 = 110$

$X_1 \ge 0, X_2 \ge 0, S_1 \ge 0, S_2 \ge 0$

We have m= 2 and n= 4 , thus n-m=2 (Non-basic variable which equal zero)

Initial Basic Feasible Solution = (0,0,300,110)

Iteration 1

Entering Variable (pivot Colum)

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	-3	-4	0	0	0	
S_1	15	10	1	0	300	300/10=30
S_2	2.5	5	0	1	110	110/5=22

Leaving Variable

$\frac{Row\ 3}{pivot\ element}$

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	-3	-4	0	0	0	
S_1	15	10	1	0	300	300/10=30
x_2	0.5	1	0	0.2	22	110/5=22

Row 2 -(10) Row 3 = new Row2

Row 1 -(-4) Row 3 = new Row1

Iteration 2

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	-1	0	0	4/5	88	
S_1	10	0	1	-2	80	80/10=8
x_2	0.5	1	0	0.2	22	22/0.5=44

Basic Variables	x_1	x_2	S_1	S_2	Solution
Z	0	0	0.1	0.6	96
x_1	1	0	0.1	-0.2	8
x_2	0	1	-0.05	0.3	18

Row 1 -(-1) (Row2 /10) = new Row1

Row 3 -(0.5) (Row2/10) = new Row3

The optimal solution: $\boldsymbol{x_1 = 8}$, $\boldsymbol{x_2 = 18}$, $\boldsymbol{S_1 = S_2 = 0}$, $\boldsymbol{Z = 96}$

2- Min $\boldsymbol{Z = -3X_1 + X_2}$

Subject to

$X_1 + X_2 \leq 5$

$2X_1 + X_2 \leq 8$

$X_1 \geq 0, X_2 \geq 0$

Solution:

The standard form of LPP

Min $\boldsymbol{Z + 3X_1 - X_2 = 0}$

Subject to

$X_1 + X_2 + S_1 = 5$

$2X_1 + X_2 + S_2 = 8$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0$

We have m= 2 and n= 4 , thus n-m=2 (Non-basic variable which equal zero)

Initial Basic Feasible Solution = (0,0,5,8)

Basic Variables	x_1	x_2	S_1	S_2	Solution	Ratio
Z	3	-1	0	0	0	
S_1	1	1	1	0	5	5/1=5
S_2	2	1	0	1	8	8/2=4

Basic Variables	x_1	x_2	S_1	S_2	Solution
Z	0	-5/2	0	-3/2	-12
S_1	0	1/2	1	-1/2	1
x_1	1	1/2	0	1/2	4

We note all coefficient of objective function are non-positive values. Thus, the optimal solution is $\boldsymbol{x_1 = 4}$, $\boldsymbol{S_1 = 1}$, $\boldsymbol{x_2 = 0}$, $\boldsymbol{S_2 = 0}$, **Z= -12**

3- Max $Z = 200X_1 + 140X_2$

Subject to

$3X_1 \leq 6000$

$2.9X_2 \leq 8000$

$2.5X_1 + 2X_2 \leq 7500$

$1.3X_1 + 1.5X_2 \leq 5000$

$X_1 \geq 0, X_2 \geq 0$

Solution: (we have canonical form)

The standard form of LPP

$$\text{Max } Z - 200X_1 - 140X_2 = 0$$

Subject to

$3X_1 + S_1 = 6000$

$2.9X_2 + S_2 = 8000$

$2.5X_1 + 2X_2 + S_3 = 7500$

$1.3X_1 + 1.5X_2 + S_4 = 5000$

$X_1 \geq 0, X_2 \geq 0, S_1 \geq 0, S_2 \geq 0, S_3 \geq 0, S_4 \geq 0$

We have m= 4 and n= 6 , thus n-m=2 (Non-basic variable which equal zero)

Iteration 1								
Basic Variables	x_1	x_2	S_1	S_2	S_3	S_4	**Solution**	**Ratio**
Z	**-200**	**-140**	**0**	**0**	**0**	**0**	**0**	
S_1	**3**	**0**	**1**	**0**	**0**	**0**	**6000**	**6000/3=2000**
S_2	**0**	**2.9**	**0**	**1**	**0**	**0**	**8000**	**-----**
S_3	**2.5**	**2**	**0**	**0**	**1**	**0**	**7500**	**7500/2.5=3000**
S_4	**1.3**	**1.5**	**0**	**0**	**0**	**1**	**5000**	**5000/1.3=3846**

New pivot row= current pivot row / pivot element
All other rows
New row= (current row) - (pivot column coefficient) (New pivot row)

Row 1	Row 3	Row 4	Row 5
[-200 -140 0 0 0 0 0] **- (-200)*** [1 0 1/3 0 0 0 2000] = [0 -140 200/3 0 0 0 400000]	[0 2.9 0 1 0 0 8000] **-(0)*** [1 0 1/3 0 0 0 2000] =[0 2.9 0 1 0 0 8000]	[2.5 2 0 0 1 0 7500] **- (2.5)*** [1 0 1/3 0 0 0 2000] =[0 2 -5/6 0 1 0 2500]	[1.3 1.5 0 0 0 1 5000] **- (1.3)*** [1 0 1/3 0 0 0 2000] =[0 1.5 -13/30 0 0 1 2400]

Iteration 2								
Basic Variables	x_1	x_2	S_1	S_2	S_3	S_4	**Solution**	**Ratio**
Z	**0**	**-140**	**200/3**	**0**	**0**	**0**	**400000**	
x_1	**1**	**0**	**1/3**	**0**	**0**	**0**	**2000**	**----**
S_2	**0**	**2.9**	**0**	**1**	**0**	**0**	**8000**	**8000/2.9=2758.62**
S_3	**0**	**2**	**-5/6**	**0**	**1**	**0**	**2500**	**2500/2=1250**
S_4	**0**	**1.5**	**-13/30**	**0**	**0**	**1**	**2400**	**2400/1.5=1600**

[0 -140 200/3 0 0 0 400000] **-(-140)*** **[0 1 -5/12 0 0.5 0 1250]** = [0 0 25/3 0 70 0 575000]	[1 0 1/3 0 0 0 2000] **-(0)*** **[0 1 -5/12 0 0.5 0 1250]** = [1 0 1/3 0 0 0 2000]	[0 2.9 0 1 0 0 8000] **-(2.9)*** **[0 1 -5/12 0 0.5 0 1250]** = [0 0 7.25/6 0 -2.9/2 0 4375]	[0 1.5 -1.3/3 0 0 1 2400] **-(1.5)*** **[0 1 -5/12 0 0.5 0 1250]** = [0 0 1.15/6 0 -1.5/2 1]

Iteration 3							
Basic Variables	x_1	x_2	S_1	S_2	S_3	S_4	**Solution**
Z	**0**	**0**	**25/3**	**0**	**70**	**0**	**575000**
x_1	**1**	**0**	**1/3**	**0**	**0**	**0**	**2000**
S_2	**0**	**0**	**7.25/6**	**1**	**-2.9/2**	**0**	**4375**
x_2	**0**	**1**	**-2.5/6**	**0**	**1/2**	**0**	**1250**
S_4	**0**	**0**	**1.15/6**	**0**	**-1.5/2**	**1**	**525**

The optimal solution:

$x_1 = 2000,\ S_2 = 4375\ ,\ x_2 = 1250\ ,\ S_4 = 525\ ,$ **Z=575000**

H.W **3- Max** $Z = 30X_1 + 20X_2 + 5\,X_3$

Subject to

$2X_1 + X_2 + X_3 \le 8$

$X_1 + 3X_2 - 4X_3 \le 8$

$X_1 \ge 0, X_2 \ge 0\ , X_3 \ge 0$

H.W 4- Max $Z = 2X_1 - X_2 + X_3$

Subject to

$2X_1 + X_2 \leq 10$

$X_1 + 2X_2 - 2X_3 \leq 20$

$X_2 + 2X_3 \leq 5$

$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0$

Solution: (we have canonical form)

The standard form of LPP

Max z

$Z - 2X_1 + X_2 - X_3 = 0$

$2X_1 + X_2 + s_1 = 10$

$X_1 + 2X_2 - 2X_3 + s_2 = 20$

$X_2 + 2X_3 + s_3 = 5$

$X_1 \geq 0, X_2 \geq 0, X_3 \geq 0, s_1, s_2 s_3 \geq 0$

We have m= 3 and n= 6 , thus n-m=3 (Non-basic variable which equal zero)

Iteration 1								
Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	**Solution**	**Ratio**
Z	-2	1	-1	0	0	0	0	
S_1	2	1	0	1	0	0	10	10/2= 5
S_2	1	2	-2	0	1	0	20	20/1= 20
S_3	0	1	2	0	0	1	5	---

Iteration 2								
Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	**Solution**	**Ratio**
Z	0	2	-1	1	0	0	10	
x_1	1	1/2	0	1/2	0	0	5	---
S_2	0	3/2	-2	-1/2	1	0	15	---
S_3	0	1	2	0	0	1	5	5/2 =2.5

Iteration 3							
Basic Variables	x_1	x_2	x_3	S_1	S_2	S_3	**Solution**
Z	0	5/2	0	1	0	1/2	25/2
x_1	1	1/2	0	1/2	0	0	5
S_2	0	5/2	0	-1/2	1	1	20
x_3	0	1/2	1	0	0	1/2	5/2

The optimal solution: $Z = \frac{25}{2}, x_1 = 5, x_2 = 0, x_3 = \frac{5}{2}, s_2 = 20, s_1 = 0, s_3 = 0$