alternate solution of **Example 2** : The quantity ε is assigned to *cell (2,4)* , which has the minimum transportation cost = **0**.

Example2: (degenerate) A company has factories at S1, S2 and S3 which supply to warehouses at D1, D2, D3 and D4. Weekly factory capacities are 18, 3 and 30 units, respectively. Weekly warehouse requirement are 21, 15, 9 and 6 units, respectively. Unit shipping costs (in Dollar) are as follows:

Destination / Sources	D_1	D_2	D_3	D_4	Supply
S_1	8	21	44	28	18
S_2	4	0	24	4	3
S_3	20	32	60	36	30
Demand	21	15	9	6	

Answer:

Destination / Sources	D_1	D_2	D_3	D_4	Supply	
S_1	8 18	21	44	28	18	0
S_2	4 3	0	24	4	3	0
S_3	20	32 15	60 9	36 6	30	15 6 0
Demand	21	15	9	6	51 51	
	3 0	0	0	0		

Initial feasible solution (IBFS) is:

$$X_{11} = 18,\ X_{21} = 3,\ X_{32} = 15,\ \ X_{33} = 9, X_{34} = 6$$

The minimum total transportation cost:

$$TTC = Z = 8 * 18 + 4 * 3 + 32 * 15 + 60 * 6 = \mathbf{1392\$}$$

Here, the number of allocated cells = 5, which is less than to **m + n - 1 = 3 + 4 - 1 = 6**

Therefore, this solution is degenerate.

The quantity **d** is assigned to that unoccupied cell, which has the minimum transportation cost.

The quantity d is assigned to *cell (2,4)* , which has the minimum transportation cost = 4.

Optimality test using MODI method... $\boldsymbol{\delta_{kj} = v_j + u_i - C_{kj}}$,

	Iteration-1	V₁= 36	V₂= 32	V₃= 60	V₄= 36	
	Destination / Sources	**D₁**	**D₂**	**D₃**	**D₄**	**Supply**
U₁= -28	**S₁**	8 18	21 δ= -17	44 δ= -12	28 δ= -20	18
U₂= -32	**S₂**	- 4 3	0 δ= 0	24 δ= 4	+ 4 d=0	3
U₃= 0	**S₃**	+ 20 δ= 16	32 15	60 9	- 36 6	30
	Demand	21	15	9	6	51 / 51

To Find u_i and v_j for all occupied cells (i, j), where $v_j + u_i = C_{ij}$

- Substituting, u_3=0, we get
- $c_{32} = u_3 + v_2 \Rightarrow v_2 = c_{32} - u_3 \Rightarrow v_2 = 32 - 0 = 32$
- $c_{33} = u_3 + v_3 \Rightarrow v_3 = c_{33} - u_3 \Rightarrow v_3 = 60 - 0 \Rightarrow v_3 = 60$
- $c_{34} = u_3 + v_4 \Rightarrow v_4 = c_{34} - u_3 \Rightarrow v_4 = 36 - 0 = 36$
- $c_{24} = u_2 + v_4 \Rightarrow u_2 = c_{24} - v_4 \Rightarrow u_2 = 4 - 36 = -32$
- $c_{21} = u_2 + v_1 \Rightarrow v_1 = c_{21} - u_2 \Rightarrow v_1 = 4 - (-32) = 36$
- $c_{11} = u_1 + v_1 \Rightarrow u_1 = c_{11} - v_1 \Rightarrow u_1 = 8 - 36 = -28$

We note that not all $\boldsymbol{\delta_{kj}} \leq 0$, so we don't reach to optimal solution yet.

	Iteration-2	V₁= 20	V₂= 32	V₃= 60	V₄= 36	
	Destination / Sources	**D₁**	**D₂**	**D₃**	**D₄**	**Supply**
U₁= -12	**S₁**	- 8 18	21 δ= -1	+ 44 δ= 4	28 δ= - 4	18
U₂= -32	**S₂**	4 δ= -16	0 δ= 0	24 δ= 4	4 3	3
U₃= 0	**S₃**	+ 20 3	32 15	- 60 9	36 3	30

Bayan Almukhlif

	Demand	**21**	**15**	**9**	**6**	51 \ 51

We note that not all $\boldsymbol{\delta_{kj}} \leq 0$, so we don't reach to optimal solution yet.

	Iteration-3	V_1= 20	V_2= 32	V_3= 56	V_4= 36	
	Destination \ Sources	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
U_1= -12	**S_1**	**8** 9	**21** δ= -1	**44** 9	**28** δ= - 4	**18**
U_2= -32	**S_2**	**4** δ= -16	**0** δ= 0	**24** δ= 0	**4** 3	**3**
U_3= 0	**S_3**	**20** 12	**32** 15	**60** δ= -4	**36** 3	**30**
	Demand	**21**	**15**	**9**	**6**	51 \ 51

We note that all $\boldsymbol{\delta_{kj}} \leq 0$, so final optimal solution is arrived

The minimum total transportation cost:

$$TTC = Z = 8(9) = 44(9) + 4(3) + 20(12) + 32(15) + 36(3) = \mathbf{1308}\$$$

Note: alternate solution is available with unoccupied cell (2,2) , but with the same optimal value.