# Exercise

***Example 1:*** An office has five workers, and five tasks have to be performed. Workers differ in efficiency and tasks differ in their intrinsic difficulty. Time each worker would take to complete each task is given in the effectiveness matrix.

How the tasks should be allocated to each worker so as to minimize the total man-hours?

| workers / tasks | **1** | **2** | **3** | **4** | **5** | **$P_i$=min** |
|---|---|---|---|---|---|---|
| **A** | 9 | 11 | 14 | 11 | 7 | **7** |
| **B** | 6 | 15 | 13 | 13 | 10 | **6** |
| **C** | 12 | 13 | 6 | 8 | 8 | **6** |
| **D** | 11 | 9 | 10 | 12 | 9 | **9** |
| **E** | 7 | 12 | 14 | 10 | 14 | **7** |

We note #Row = # column, we can solve this problem.

| | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **A** | **2** | **4** | **7** | **4** | **0** |
| **B** | **0** | **9** | **7** | **7** | **4** |
| **C** | **6** | **7** | **0** | **2** | **2** |
| **D** | **2** | **0** | **1** | **3** | **0** |
| **E** | **0** | **5** | **7** | **3** | **7** |
| **$q_j$=min** | **0** | **0** | **0** | **2** | **0** |

| | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **A** | **2** | **4** | **7** | **2** | **0** |
| **B** | **0** | **9** | **7** | **5** | **4** |
| **C** | **6** | **7** | **0** | **0** | **2** |
| **D** | **2** | **0** | **1** | **1** | **0** |
| **E** | **0** | **5** | **7** | **1** | **7** |

$since\ \#raw\ or\ colum \neq k;$

$so\ determain\ h = \min\{cell(i,j) is\ not\ coverd\} = 1$

| | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| **A** | **2** | **4** | **6** | **1** | **0** step 1 |
| **B** | **0** step 2 | **9** | **6** | **4** | **4** |
| **C** | **7** | **8** | **0** step 4 | **0** | **3** |
| **D** | **2** | **0** step 5 | **0** | **0** | **0** |
| **E** | **0** | **5** | **6** | **0** step 3 | **7** |

$x_{15} = x_{21} = x_{54} = x_{33} = x_{42} = 1$

Optimum solution

| Man | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Job | B | D | C | E | A |
| Man hours | 6 | 9 | 6 | 10 | 7 |

$Total\ time = Z^* = 38\ hours$

**Step 1:** In a given problem, if the number of rows is not equal to the number of columns and vice versa, then add a dummy row or a dummy column. The assignment costs for dummy cells are always assigned as zero.

**Step 2:** Row reduction: Subtract the smallest element in each row from all elements in the respective row .

**Step 3:** Column reduction.

**Step 4:** Draw minimum number of lines to cover all zeros.

**Step 5:** If (k) Number of lines drawn = order of matrix then optimally is reached, so proceed to step 7. If optimally is not reached, then go to step 6

**Step 6:** Select the smallest element , which is NOT COVERED by lines. Then, subtract this smallest element from all the uncovered elements and add it to the elements at intersection of the lines. Now go to step 4.

**Step 7:** Find allocation: start the assignment from the row or column that has minimum number of Zeros. Then,eliminate row and column related to that assignment.

**Step 8:** Write down the assignment results and find the minimum cost/time.

***Example 2:*** A work shop contains four persons available for work on the four jobs. Only one person can work on any one job. The following table shows the cost of assigning each person to each job. The objective is to assign person to jobs such that the total assignment cost is a minimum.

| Jobs | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| **I** | 20 | 25 | 22 | 28 |
| **II** | 15 | 18 | 23 | 17 |
| **III** | 19 | 17 | 21 | 24 |
| **IV** | 25 | 23 | 24 | 24 |

Solution:

Find the First Reduced Cost Table:

| Jobs | **1** | **2** | **3** | **4** | **$P_i$=min** |
|---|---|---|---|---|---|
| **I** | 20 | 25 | 22 | 28 | 20 |
| **II** | 15 | 18 | 23 | 17 | 15 |
| **III** | 19 | 17 | 21 | 24 | 17 |
| **IV** | 25 | 23 | 24 | 24 | 23 |

Find the Second Reduced Cost Table:

| Jobs / Persons | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| **I** | 0 | 5 | 2 | 8 |
| **II** | 0 | 3 | 8 | 2 |
| **III** | 2 | 0 | 4 | 7 |
| **IV** | 2 | 0 | 1 | 1 |
| **$q_j$=min** | 0 | 0 | 1 | 1 |

Draw minimum number of lines to cover all zeros:

| Jobs / Persons | **1** | **2** | **3** | **4** |
|---|---|---|---|---|
| **I** | 0 | 5 | 1 | 7 |
| **II** | 0 | 3 | 7 | 1 |
| **III** | 2 | 0 | 3 | 6 |
| **IV** | 2 | 0 | 0 | 0 |

$since \ \#raw \ or \ colum \neq k = 4;$
$so \ determain \ h = \min\{cell(i,j) is \ not \ coverd\} = 1$

| Jobs / Persons | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| I | 0 | 4 | 0 | 6 |
| II | 0 | 2 | 6 | 0 |
| III | 3 | 0 | 3 | 6 |
| IV | 3 | 0 | 0 | 0 |

$since \ \#raw \ or \ colum = k = 4$ so, we arrive at an optimal solution (assignment).
Determine an assignment:

| Jobs / Persons | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| I | 0 [20] | 4 | 0 step 3 [22] | 6 |
| II | 0 step 2 [15] | 2 | 6 | 0 [17] |
| III | 3 | 0 step 1 | 3 | 6 |
| IV | 3 | 0 | 0 [24] | 0 step 4 [24] |

Optimum solution

| Persons | I | II | III | IV |
|---|---|---|---|---|
| Job | 3 | 1 | 2 | 4 |
| Man hours | 22 | 15 | 17 | 24 |

$Total \ time = Z^* = 78$

***Example 3:*** There are five jobs to be assigned to the machines. Only one job could be assigned to one machine are given in following matrix.

| Machines / Jobs | A | B | C | D |
|---|---|---|---|---|
| 1 | 1 | 6 | 4 | 3 |
| 2 | 0 | 7 | 2 | 1 |
| 3 | 3 | 7 | 2 | 4 |
| 4 | 4 | 6 | 5 | 7 |
| 5 | 3 | 2 | 4 | 6 |

1- Find an optimum assignment of jobs to the machines to minimize the total processing time.( using Hungarian method)

2- Find for which job no machine is assigned.
3- What is the total processing time to complete all the jobs.

## Solution:

We note #Row ≠ # column, so we need to add dummy column (machine).

| | A | B | C | D | Dummy |
|---|---|---|---|---|---|
| **1** | 1 | 6 | 4 | 3 | 0 |
| **2** | 0 | 7 | 2 | 1 | 0 |
| **3** | 3 | 7 | 2 | 4 | 0 |
| **4** | 4 | 6 | 5 | 7 | 0 |
| **5** | 3 | 2 | 4 | 6 | 0 |
| **$q_k$** | **0** | **2** | **2** | **1** | **0** |

| | A | B | C | D | Dummy |
|---|---|---|---|---|---|
| **1** | **1** | **4** | **2** | **2** | **0** |
| **2** | **0** | **5** | **0** | **0** | **0** |
| **3** | **3** | **5** | **0** | **3** | **0** |
| **4** | **4** | **4** | **3** | **6** | **0** |
| **5** | **3** | **0** | **2** | **5** | **0** |

$h = 1$

| | A | B | C | D | Dummy |
|---|---|---|---|---|---|
| **1** | **0** step 3 | **4** | **2** | **1** | **0** |
| **2** | **0** | **6** | **1** | **0** step 4 | **1** |
| **3** | **2** | **5** | **0** step 5 | **2** | **0** |
| **4** | **3** | **4** | **3** | **5** | **0** step 1 |
| **5** | **2** | **0** step 2 | **2** | **4** | **0** |

$$x_{45} = x_{52} = x_{33} = x_{24} = x_{11} = 1$$

Optimum solution

| Jobs | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Machines | A | D | C | dummy | B |
| hours | 1 | 1 | 2 | - | 2 |

For job 4 no machines is assigned

$Total\ time = Z^* = 6\ hours$

**H.W**

***Example 4:*** There are four jobs to be assigned to the machines. Only one job could be assigned to one machine are given in following matrix.

| Jobs | Machines | | | | |
|---|---|---|---|---|---|
| | *A* | *B* | *C* | *D* | *E* |
| 1 | 4 | 3 | 6 | 2 | 7 |
| 2 | 10 | 12 | 11 | 14 | 16 |
| 3 | 4 | 3 | 2 | 1 | 5 |
| 4 | 8 | 7 | 6 | 9 | 6 |

1- Find an optimum assignment of jobs to the machines to minimize the total processing time.( using Hungarian method)
2- find for which machine no job is assigned.
3- What is the total processing time to complete all the jobs?

**Solution:**

| Job | Machine |
|---|---|
| 1 | B |
| 2 | A |
| 3 | D |
| 4 | C |

For machines E no job is assigned

Total time = 10 + 3 + 6 + 1= 20

***Example 5 :*** Assign the three tasks (A,B,C) to three operators(using Hungarian method). The assigning costs are given in Table

| | A | B | C |
|---|---|---|---|
| 1 | 20 | 15 | 31 |
| 2 | 17 | 16 | 33 |
| 3 | 18 | 19 | 27 |

**Solution:**

**Example 6:**

A dairy plant has five milk tankers I, II, III, IV & V. These milk tankers are to be used on five delivery routes A, B, C, D, and E. The distances (in kms) between dairy plant and the delivery routes are given in the following distance matrix

| | **I** | **II** | **III** | **IV** | **V** |
|---|---|---|---|---|---|
| **A** | **160** | **130** | **175** | **190** | **200** |
| **B** | **135** | **120** | **130** | **160** | **175** |
| **C** | **140** | **110** | **155** | **170** | **185** |
| **D** | **50** | **50** | **80** | **80** | **110** |
| **E** | **55** | **35** | **70** | **80** | **105** |

How the milk tankers should be assigned to the chilling centers so as to minimize the distance travelled?