## Example 9

## \#Decision Tree Analysis

## Example 1: A bicycle shop

Ahmad and Khaled run a small bicycle shop called "Z to A Bicycles". They must order bicycles for the coming season. Orders for the bicycles must be placed in quantities of twenty (20). The cost per bicycle is $\$ 70$ if they order 20, $\$ \mathbf{6 7}$ if they order $\mathbf{4 0}, \$ \mathbf{6 5}$ if they order $\mathbf{6 0}$, and $\$ \mathbf{6 4}$ if they order $\mathbf{8 0}$. The bicycles will be sold for $\$ \mathbf{1 0 0}$ each. Any bicycles left over at the end of the season can be sold (for certain) at $\$ 45$ each. If Ahmad and Khaled run out of bicycles during the season, then they will suffer a loss of "goodwill" among their customers. They estimate this goodwill loss to be $\$ \mathbf{5}$ per customer who was unable to buy a bicycle. Ahmad and Khaled estimate that the demand for bicycles this season will be $\mathbf{1 0}, \mathbf{3 0}, \mathbf{5 0}$, or $\mathbf{7 0}$ bicycles with probabilities of $\mathbf{0 . 2 , 0 . 4}$, $\mathbf{0 . 3}$, and 0.1 respectively.

- Create a payoff table.
- Draw a decision tree in order to determine the best strategy.


## Actions

There are four actions available to Ahmad and Khaled. They have to decide which of the actions is the best one under each criteria.

1. Buy 20 bicycles
2. Buy 40 bicycles
3. Buy 60 bicycles
4. Buy 80 bicycles

Ahmad and Khaled have control over which action they choose. That is the whole point of decision theory - deciding which action to take.

## States of Nature

There are four possible states of nature. A state of nature is an outcome.

1. The demand is 10 bicycles
2. The demand is 30 bicycles
3. The demand is 50 bicycles
4. The demand is 70 bicycles

Ahmad and Khaled have no control over which state of nature will occur. They can only plan and make the best decision based on the appropriate decision criteria.

## Payoff Table

After deciding on each action and state of nature, create a payoff table. The numbers in parentheses for each state of nature represent the probability of that state occurring.

|  | Action |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| State of Nature | Buy 20 | Buy 40 | Buy 60 | Buy 80 |
| Demand 10 <br> $(0.2)$ | $\mathbf{5 0}$ | -330 | -650 | -970 |
| Demand 30 <br> $(0.4)$ | 550 | $\mathbf{7 7 0}$ | 450 | 130 |
| Demand 50 <br> $(\mathbf{0 . 3})$ | 450 | 1270 | $\mathbf{1 5 5 0}$ | 1230 |
| Demand 70 <br> $\mathbf{( 0 . 1 )}$ | 350 | 1170 | 2050 | $\mathbf{2 3 3 0}$ |

Ok, the question on your mind is probably "How did you come up with those numbers?" Let's take a look at a couple of examples.

Demand is 50, buy 60:
They bought 60 at $\$ 65$ each for $\$ 3900$. That is $-\$ 3900$ since that is money they spent. Now, they sell 50 bicycles at $\$ 100$ each for $\$ 5000$. They had 10 bicycles left over at the end of the season, and they sold those at $\$ 45$ each of $\$ 450$. That makes $\$ 5000+450-$ $3900=\$ 1550$.

Demand is 70, buy 40:
They bought 40 at $\$ 67$ each for $\$ 2680$. That is a negative $\$ 2680$ since that is money they spent. Now, they sell 40 bicycles (that's all they had) at $\$ 100$ each for $\$ 4000$. The other 30 customers that wanted a bicycle, but couldn't get one, left mad and Ahmad and Khaled lost $\$ 5$ in goodwill for each of them. That's 30 customers at $-\$ 5$ each or $-\$ 150$. That makes $\$ 4000-2680-150=\$ 1170$.


The best strategy is Buy 40 bicycles.

Example 2: Geoffrey Ramsbottom runs a kitchen that provides food for various canteens throughout a large organization. A particular salad is sold to the canteen for $\$ 10$ and costs $\$ 8$ to prepare. Therefore, the contribution per salad is $\$ 2$.

Based upon past demands, it is expected that, during the 250 days working year, the canteens will require the following daily quantities:

- On 25 days of the year, 40 salads.
- On 50 days of the year, 50 salads.
- On 100 days of the year, 60 salads.
- On 75 days 70 salads.

The kitchen must prepare the salads in batches of 10 meals in advance. The manager has asked you to help decide how many salads the kitchen should supply for each day of the forthcoming year.

## Constructing a payoff table:

If 40 salads will be required on 25 days of a 250-day year, then the probability that demand $=40$ salads is
$\mathrm{P}($ Demand of 40$)=25$ days $\div 250$ days $=0.10$
Likewise,

- $\mathrm{P}($ Demand of 50$)=50 \div 250=0.20$
- $P($ Demand of 60$)=100 \div 250=0.40$
- $\mathrm{P}($ Demand of 70$)=75 \div 250=0.30$

Now let's look at the different values of profit or losses depending on how many salads are supplied and sold.

For example, if we supply 40 salads and all are sold, our profits amount to $40 \times \$ 2=80$. If however we supply 50 salads but only 40 are sold, our profits will amount to $40 \times \$ 2$ - (10 unsold salads $\times \$ 8$ unit cost) $=80-80=0$.

Similarly, we can now construct a payoff table as follows:

| Daily supply |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Probability | $\mathbf{4 0}$ salads | $\mathbf{5 0}$ salads | $\mathbf{6 0}$ salads | $\mathbf{7 0}$ salads |  |
| Daily Demand | 40 salads | 0.10 | $\$ 80$ | $\$ 0$ | $\$(-80)$ | $\$(-160)$ |  |
|  | $\mathbf{5 0}$ salads | 0.20 | $\$ 80$ | $\$ 100$ | $\$ 20$ | $\$(-60)$ |  |
|  | $\mathbf{6 0}$ salads | 0.40 | $\$ 80$ | $\$ 100$ | $\$ 120$ | $\$ 40$ |  |
|  | 70 salads | 0.30 | $\$ 80$ | $\$ 100$ | $\$ 120$ | $\$ 140$ |  |



The best strategy is supply $\mathbf{5 0}$ salads for each day of the forthcoming year.
H.W

Example : A property owner is faced with a choice of:
a) A large-scale investment (A) to improve her flats. This could produce a substantial payoff in terms of increased revenue net of costs but will require an investment of $\mathbf{£ 1 , 4 0 0 , 0 0 0}$. After extensive market research it is considered that there is a $\mathbf{4 0 \%}$ chance that a pay-off of $£ 2,500,000$ will be obtained, but there is a $\mathbf{6 0 \%}$ chance that it will be only $£ 800,000$.
b) A smaller scale project (B) to re-decorate her premises. At $\mathbf{£ 5 0 0 , 0 0 0}$ this is less costly but will produce a lower pay-off. Research data suggests a $\mathbf{3 0 \%}$ chance of a gain of $\mathbf{£ 1 , 0 0 0 , 0 0 0}$ but a $\mathbf{7 0 \%}$ chance of it being only $\mathbf{£ 5 0 0 , 0 0 0}$.
c) Continuing the present operation without change ( $\mathbf{C}$ ). It will cost nothing, but neither will it produce any pay-off. Clients will be unhappy and it will become harder and harder to rent the flats out when they become free.

How will a decision tree help the taking of the decision?

