

Final Dec19 Grading scheme

Question 1

$$a) \int_0^3 (4x - x^2) dx = 3(4c - c^2) \Leftrightarrow c^2 - 4c + 3 = 0 \quad (1.5)$$

$$c = 1 \text{ or } c = 3 \quad (0,5)$$

$$b) \int \frac{dx}{\sqrt{5x-16}} = \frac{2}{\ln 5} \int \frac{du}{u\sqrt{u^2-16}} = \frac{1}{2\ln 5} \sec^{-1} \left(\frac{5x}{4} \right) + C \quad (1.5) + (0.5)$$

$$c) \int \frac{\cot x dx}{\sqrt{9-(\sin x)^4}} = \frac{1}{2} \int \frac{du}{u\sqrt{9-u^2}} \quad u = (\sin x)^2 \quad (2)$$
$$= \frac{-1}{6} \operatorname{sech}^{-1} \left(\frac{(\sin x)^2}{3} \right) + C \quad (1)$$

Question 2

$$a) \lim_{x \rightarrow 3} \frac{\ln(x-2) - x + 3}{(x-3)\ln(x-2)} = \lim_{x \rightarrow 3} \frac{\frac{1}{x-2} - 1}{\ln(x-2) + \frac{x-3}{x-2}} = \lim_{x \rightarrow 3} \frac{\frac{-1}{(x-2)^2}}{\frac{1}{x-2} + \frac{1}{(x-2)^2}}$$
$$= -\frac{1}{2} \quad (1) + (1) + (1)$$

$$b) \int x^2 \tan^{-1}(x) dx = \frac{x^3}{3} \tan^{-1}(x) - \frac{1}{3} \int \frac{x^3}{1+x^2} dx$$
$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{3} \int \left(x - \frac{x}{1+x^2} \right) dx$$
$$= \frac{x^3}{3} \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2) + C$$

$$(1) + (1) + (1)$$

$$c) \int (\tan x)^4 (\sec x)^6 dx = \int u^4 (u^2 - 1)^2 du$$
$$= \frac{(\tan x)^9}{9} - \frac{2}{7} (\tan x)^7 + \frac{1}{5} (\tan x)^5 + C$$
$$(1.5) + (1.5)$$

Question 3

$$a) \int \frac{x^2 dx}{(x^2+9)^{3/2}} = \int \frac{(\tan\theta)^2}{\sec(\theta)} d\theta = \int (\sec\theta - \cos\theta) d\theta \quad x = 3\tan\theta \quad (1.5)$$

$$= \ln \left| \frac{\sqrt{x^2+9}}{3} + \frac{x}{3} \right| - \frac{x}{\sqrt{x^2+9}} + C \quad (1.5)$$

$$b) \frac{3x-2}{(x+2)(x^2+4)} = \frac{-1}{x+2} + \frac{x+1}{x^2+4} \quad (1.5)$$

$$\int \frac{(3x-2)dx}{(x+2)(x^2+4)} = -\ln|x+2| + \frac{1}{2}\ln(1+x^2) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C \quad (1.5)$$

$$d) \int \frac{dx}{3-\sin x+\cos x} = \frac{1}{2} \int \frac{du}{2-u+u^2} = \frac{1}{2} \int \frac{du}{(u-\frac{1}{2})^2+\frac{7}{4}} \quad (2)$$

$$= \frac{1}{\sqrt{7}} \tan^{-1}\left(\frac{2u-1}{\sqrt{7}}\right) + C = \frac{1}{\sqrt{7}} \tan^{-1}\left(\frac{2\tan(\frac{x}{2})-1}{\sqrt{7}}\right) + C \quad (1)$$

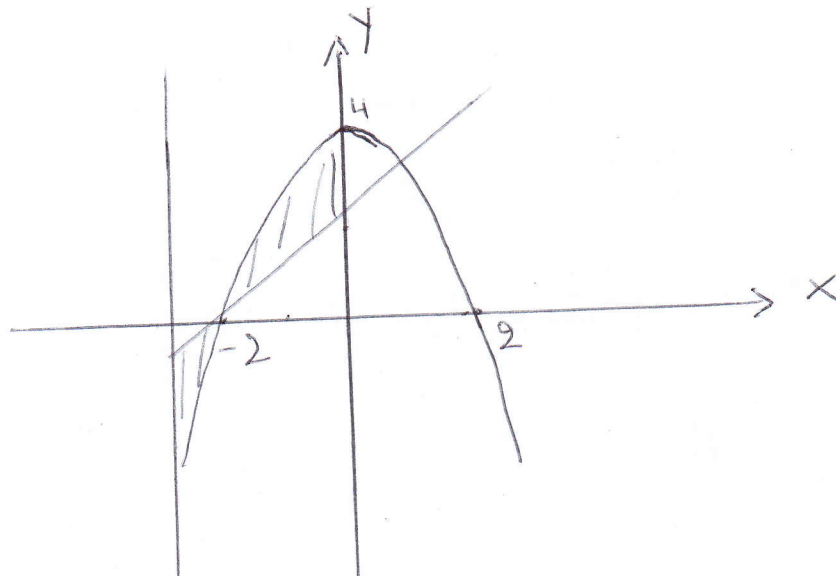
Question 4

$$a) x+2 = 4-x^2 \Leftrightarrow x = -2 \text{ or } x = 1$$

$$A = \int_{-3}^{-2} x+2 - (4-x^2) dx + \int_{-2}^0 4-x^2 - (x+2) dx \quad (1.5)$$

$$= \frac{11}{6} + \frac{10}{3} = \frac{31}{6} \quad (0.5)$$

graph (1)



$$b) V = \int_{-1}^1 2\pi(3-x)(-x^2+1)dx = 8\pi \quad (2)$$

$$c) V = \int_{-1}^1 \pi(9 - (2+x^2)^2)dx \quad (1)$$

Question 5

$$a) L = \int_0^\pi \sqrt{(\cos(\frac{\theta}{2}))^4 + (\cos(\frac{\theta}{2}))^2(\sin(\frac{\theta}{2}))^2} d\theta \quad (1.5)$$

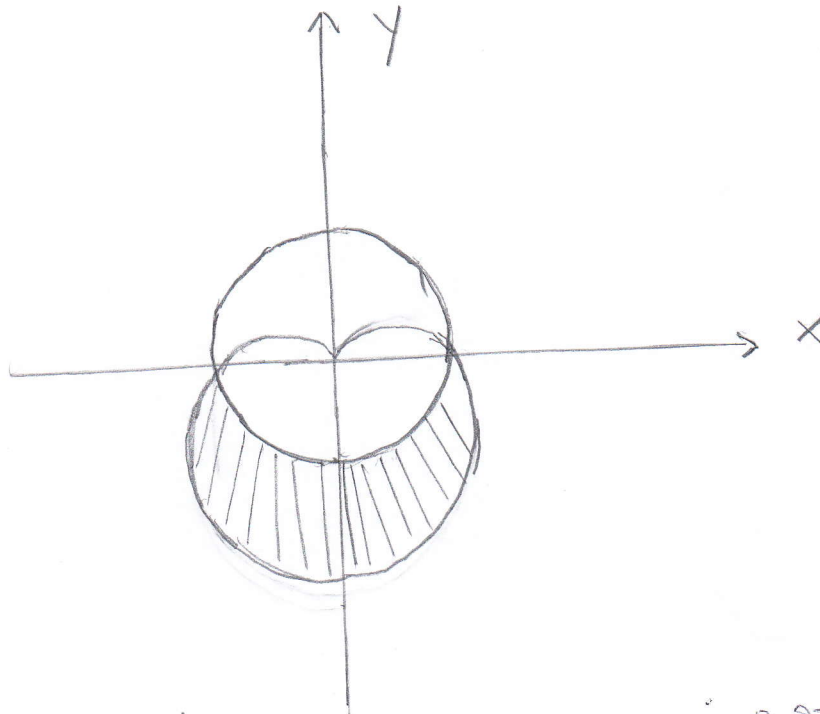
$$= \int_0^\pi \cos(\frac{\theta}{2}) d\theta = 2 \quad (1.5)$$

$$b) 1 - \sin\theta = 1 \Rightarrow \theta = 0 \text{ or } \theta = \pi \text{ or } \theta = 2\pi$$

$$A = \frac{1}{2} \int_\pi^{2\pi} [(1 - \sin\theta)^2 - 1] d\theta \quad (1)$$

$$= \frac{\pi}{4} + 2 \quad (1)$$

graph (1)



$$c) S = 8\pi \int_0^{\pi/4} \cos^2 \theta d\theta = 4\pi \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/4}$$

$$= 2\pi \left(\frac{\pi}{2} + 1 \right) \quad (1.5) + (1.5)$$

$$\approx \underline{\underline{0.64269}}$$