KING SAUD UNIVERSITY, DEPARTMENT OF MATHEMATICS
MATH 204. TIME: 3H, FULL MARKS: 40, FINAL EXAM

Question 1. [4,4,5] a) Solve the initial value problem

$$\begin{cases} (x + ye^{y/x})dx - xe^{y/x}dy = 0 \\ y(1) = 0 \end{cases}$$

b) Solve the differential equation $y(e^{-2x} + y^2)dx - e^{-2x}dy = 0$.

c) A thermometer is taken from an inside room to the outside, where the air temperature is 5^0F. After 1 minute the temperature reads 55^0F, and after 5 minutes it reads 30^0F. What is the initial temperature of the inside room.

Question 2. [4,5] a) If $y_1 = x^{-1}$ is a solution of the differential equation $x^2y'' + xy' - y = 0$, $x > 0$,use reduction of order to solve the differential equation

$$x^2y'' + xy' - y = \ln x, \quad x > 0.$$

b) Find the largest interval for which the following initial value problem admits a unique solution

$$\begin{cases} \frac{y''}{x^2-1} + (\tan x)y = e^x \\ y(0) = 1, \; y'(0) = 0 \end{cases}$$

Question 3. [3,5] a) Determine the form of the particular solution of the following differential equation

$$y''' - 3y' + 2y = x^2e^x + 3e^{-x} + \sin 2x.$$

b) Use power series method to find the first four nonzero terms of the solution of the initial value problem

$$\begin{cases} y'' + 3xy' - y = 0 \\ y(0) = 2, \quad y'(0) = 0 \end{cases}$$

Question 4. [5,5] a) Consider the $2\pi-$periodic even function defined by

$$f(x) = 1 - \frac{2x}{\pi}, \text{ for } x \in [0, \pi].$$

Sketch the graph of f on $(-3\pi, 3\pi)$, obtain the Fourier series for the function f, and deduce the value of the numerical series $\sum\limits_{n=0}^{\infty} \frac{1}{(2n+1)^2}$, and $\sum\limits_{n=1}^{\infty} \frac{1}{n^2}$.

b) Consider the function

$$f(x) = \begin{cases} 1 + x, & -1 \le x \le 0, \\ 1 - x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Sketch the graph of f, find the Fourier integral representation of f, and deduce the value of $\int\limits_0^\infty \frac{\sin^2 \lambda}{\lambda^2} d\lambda$.