

**King Saud University, Department of Mathematics**  
**Math 204 (3H), 40/40, Alternative Exam 20/12/2018**

**Question 1[4,4] a)** Obtain the general solution of the following Bernoulli differential equation

$$(x^3 + y^2 + 3xy^2)dx - 2xydy = 0, \quad x > 0, y \neq 0.$$

**b)** Solve the initial value problem

$$\begin{cases} 2e^x \cos y dx + (1 + e^x) \sin y dy = 0, & 0 < y < \frac{\pi}{2} \\ y(0) = \frac{\pi}{4}. \end{cases}$$

**Question 2[4,4] a)** Find the largest interval for which the following initial value problem admits a unique solution

$$\begin{cases} \sqrt[3]{x-3}y'' + (x-1)^{-1/2}y' + e^xy = (x-1)^2 \\ y(2) = -1, \quad y'(2) = 0. \end{cases}$$

**b)** Find the family of orthogonal trajectories of the family of curves

$$\sqrt{x} + \sqrt{y-C} = 1.$$

**Question 3[4,5,5] a)** Show that the functions:  $f_1(x) = x$ ,  $f_2(x) = x-1$ ,  $f_3(x) = x+3$  are linearly dependent or linearly independent on  $\mathbb{R}$ .

**b)** Use power series method to find a series solution of the differential equation

$$y'' - xy = 0,$$

about the ordinary point  $x = 0$ , which satisfies  $y(0) = 2$ ,  $y'(0) = 3$ .

**c)** Obtain the general solution of

$$x^2y'' - 3xy' + 3y = x^4, \quad x > 0.$$

**Question 4[5,5] a)** Let  $f$  be a periodic function of period  $2\pi$  given by:

$$f(x) = \pi - |x| \quad \text{for } x \in [-\pi, \pi].$$

Sketch the graph of  $f$  on  $(-3\pi, 3\pi)$ , find its Fourier series and deduce the value of the series  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ .

**b)** Sketch the graph of the following function and find its Fourier integral

$$f(x) = \begin{cases} \cos x & \text{if } |x| \leq \pi \\ 0, & \text{if } |x| > \pi \end{cases} \quad \text{Deduce that } \int_0^{\infty} \frac{\lambda \sin(\lambda\pi)}{1-\lambda^2} d\lambda = \frac{\pi}{2}.$$

(Hint:  $\sin(\pi - x) = \sin x$ ,  $\sin(\pi + x) = -\sin x$ ).