College of Sciences
Department of Mathematics

Final Exam

Math 550

Exercise 1. State whether the following are true or false. Justify your answer.

1. Let p> 0. The application $N : \mathbb{R}^n \to \mathbb{R}_+$, defined by

$$N(x) = \left(\sum_{k=1}^{n} x_k^p \right)^{\frac{1}{p}}$$

is a norm on $\mathbb{R}^n$.

2. Let $n > 2$. Every matrix norm N is a sub-multiplicative norm, that is :

$$N(AB) \leq N(A)N(B), \quad \forall\ A,\ B \in \mathbb{R}^{n\times n}$$

.

3. The floating-point form of $x = -0,00456296748$ using five-digit rounding is $fl(x) = -0,00456$.
4. If the guess (initial point) is close enough to a root of the function, then Newton's method converge quadratically.
5. The graphical interpretation of the Secant method is as follows : Starting with the initial approximation x_0, the approximation x_1 is the x-intercept of the tangent line to the graph of f at $(x_0, f(x_0))$. The approximation x_2 is the x-intercept of the tangent line to the graph of f at $(x_1, f(x_1))$ and so on.

Exercise 2.

Use three steps of the Newton's method to approximate the roots of the following system

$$\begin{cases} x + e^{-x} + y^3 = 0, \\ x^2 + 2xy - y^2 + \tan x = 0 \end{cases}$$

by taking initial guess $x_0 = 3, y_0 = -1.5$.

Exercise 3.

1. Show that the equation $e^{x^2} = 4x^2$ has a solution α in the interval $[0, 1]$.

2. Use three iterations of the bisection method in the interval $[0, 1]$ and Newton's method with initial point $x_0 = 0.5$, to approximate the zero of the function $f(x) = e^{x^2} - 4x^2$.

3. Prove that the Newton's method converge quadratically to α.

Exercise 4. Let $P(x) = x^5 - 5x^3 + 4x$.

1. Find the Sturm sequence of P .

2. Determine the number of roots of P in the interval $[-3, 3]$.

3. Determine the number of positive roots of P.