King Saud University: Mathematics Department Math-254
Third Semester 1444 H Final Examination
Maximum Marks = 40 Time: 180 mins.

Name of the Student:———————————— **I.D. No.** ——————————

Name of the Teacher:———————————— **Section No.** ——————————

Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

The Answer Tables for Q.1 to Q.15 : Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

Q. No.	1	2	3	4	5	6	7	8	9	10
a,b,c,d										

Q. No.	11	12	13	14	15
a,b,c,d					

Quest. No.	Marks Obtained	Marks for Questions
Q. 1 to Q. 15		30
Q. 16		5
Q. 17		5
Total		40

Question 1: The number of bisections required to solve the equation $x^3 - 2x = 1$ in $[1.5, 2]$ accurate to within 10^{-4} is:

(a) 13 (b) 11 (c) 15 (d) None of these

Question 2: Given $x_0 = 0$ and $x_1 = 0.1$, then the next approximation x_2 of the solution of the reciprocal of 5 using the Secant method is:

(a) 0.15 **(b)** 0.1 **(c)** 0.175 **(d)** None of these

Question 3: The order of convergence of the Newton's method for $f(x) = \tan x$ at the root $\alpha = \pi$ is:

(a) 2 **(b)** 1 **(c)** 3 **(d)** None of these

Question 4: The l_∞-norm of the inverse of the Jacobian matrix of the nonlinear system $x^2 + y^2 = 1,\ xy = 1$ at the point $(1, 0)$ is:

(a) 2 **(b)** 1 **(c)** 0.5 **(d)** None of these

Question 5: In the LU factorization with Doolittles method of the matrix $A = \begin{pmatrix} 1 & -1 \\ \alpha & 1 \end{pmatrix}$, the matrix U is singular if α is equal to:

(a) ± 1 **(b)** 1 **(c)** -1 **(d)** None of these

Question 6: The first approximation for solving linear system $A\mathbf{x} = [1, 3]^T$ using Jacobi iterative method wit $A = \begin{pmatrix} -4 & 5 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{x}^{(0)} = [0.5, 0.5]^T$ is:

(a) $[0.375, 1.250]^T$ **(b)** $[1.375, 1.315]^T$ **(c)** $[1.375, 1.250]^T$ **(d)** None of these

Question 7: Solving linear system $A\mathbf{x} = [4, 5]^T$, with $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, by Gauss-Seidel iterative method, if $\|\mathbf{x}^{(1)} - \mathbf{x}^{(0)}\| = 0.75$, then the number of iterations needed to get an accuracy within 10^{-2} is:

(a) 10 **(b)** 6 **(c)** 8 **(d)** None of these

Question 8: If $\hat{x} = [1.01, 0.99]^T$ is an approximate solution for the system of two linear equations $2x - y = 1$ and $x + y = 2$, then the error bound for the relative error is:

(a) 0.045 **(b)** 0.035 **(c)** 0.025 **(d)** None of these

Question 9: Using data points: $(0, -2), (0.1, -1), (0.15, 1), (0.2, 2), (0.3, 3)$, the best approximate value of $f(0.25)$ by a linear spline function is:

(a) 1.5 **(b)** 2.5 **(c)** 3.5 **(d)** None of these

Question 10: If $f(x) = x^2 e^x$, then $f[1, 1, 2]$ equals to:

(a) $4e^2 + 4e$ **(b)** $4e^2 - 4e$ **(c)** $4e^2 - 3e$ **(d)** None of these

Question 11: Using data points: $(0, -2), (0.1, -1), (0.15, 1), (0.2, 2), (0.3, 3)$, the best approximation of $f'(0.25)$ using 3-point difference formula is:

(a) 10.0 **(b)** 20.0 **(c)** 15.0 **(d)** None of these

Question 12: Using data points: $(0, -2), (0.1, -1), (0.15, 1), (0.2, 2), (0.3, 3)$, then the worst approximation of $f''(0.15)$ using 3-point difference formula is:

(a) -44.44 **(b)** -6.67 **(c)** -3.33 **(d)** None of these

Question 13: Using data points: $(0, -2), (0.1, -1), (0.15, 1), (0.2, 2), (0.3, 3)$, the best approximate value of the integral $\int_0^{0.3} f(x)\, dx$, using the composite Trapezoidal rule is:

(a) 0.25 **(b)** 0.1 **(c)** 0.15 **(d)** None of these

Question 14: If $f(0) = 3, f(1) = \frac{\alpha}{2}, f(2) = \alpha$, and the Simpson's rule for $\int_0^2 f(x)\, dx = 4$, then the value of α is:

(a) 1.5 **(b)** 2.0 **(c)** 3.0 **(d)** None of these

Question 15: Given $xy' + y = 1,\ y(1) = 0$, the approximate value of $y(2)$ using Euler's method when $n = 1$ is:

(a) 1.5 **(b)** 1.0 **(c)** 2.0 **(d)** None of these

Question 16: Use the following table to find the best approximation of $f(0.6)$ by using quadratic Lagrange interpolating polynomial for equally spaced data points:

x	0.15	0.2	0.3	0.5	0.55	0.8	1
$f(x)$	-0.0427	-0.0644	-0.1084	-0.1733	-0.1808	-0.1428	0

The function tabulated is $f(x) = x^2 \ln x$. Compute the absolute error and an error bound (using error bound formula for equally spaced data points) for the approximation.

Question 17: Use best integration rule to find the absolute error for the approximation of $\int_0^{1.2} f(x)\,dx$ by using the following set of data points:

x	0.0	0.1	0.21	0.3	0.42	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
$f(x)$	1.00	1.10	1.19	1.26	1.33	1.38	1.43	1.47	1.51	1.52	1.54	1.55	1.56

The function tabulated is $f(x) = x + \cos x$. How many points approximate the given integral to within accuracy of 10^{-6} ?