College of Sciences
Department of Mathematics

First Mid Term

Math 550

Exercise 1. Let $n \geq 1$.

1. Prove that
$$xy \leq \frac{x^2}{2} + \frac{y^2}{2},$$
for all $x, y \geq 0$.
2. Deduce that for all $a = (a_1, ..., a_n),\ b = (b_1, ..., b_n) \in \mathbb{R}^n$
$$\left|\sum_{k=1}^{n} a_k b_k\right| \leq \left(\sum_{k=1}^{n} a_k^2\right)^{\frac{1}{2}} \left(\sum_{k=1}^{n} b_k^2\right)^{\frac{1}{2}}$$
$$\left(\sum_{k=1}^{n} (a_k + b_k)^2\right)^{\frac{1}{2}} \leq \left(\sum_{k=1}^{n} a_k^2\right)^{\frac{1}{2}} + \left(\sum_{k=1}^{n} b_k^2\right)^{\frac{1}{2}}.$$
3. Let $x = (x_1, ..., x_n) \in \mathbb{R}^n$. Prove that $N_2(x) = \left(\sum_{k=1}^{n} x_k^2\right)^{\frac{1}{2}}$ is a norm on $\mathbb{R}^n$.

Exercise 2.

1. The number π has an infinite decimal expansion of the form $\pi = 3.14159265...$
   (a) Find the floating-point form of π using five-digit chopping.
   (b) Find the floating-point form of π using five-digit rounding.
2. Suppose that $fl(y)$ is a k-digit chopping approximation to y. Show that
$$\left|\frac{y - fl(y)}{y}\right| \leq 10^{-k+1}.$$

Exercise 3.

1. Show that the equation $\sin(x) = 0.8x$ has a solution in the interval $[1, 1.5]$.

2. Use four steps of the Bisection method to find an approximation of the root of the equation $\sin(x) = 0.8x$ starting with the interval $[1, 1.5]$.

3. How many bisection iterations will be required to locate this root in the interval $[1, 1.5]$ to accuracy of $\varepsilon = 10^{-3}$.