

1. $f(x) = \sin^4(x)$, $n = 6$

$$S_6 = \frac{2\pi - 0}{3.6} \left(0 + 4 \sin^4 \frac{\pi}{3} + 2 \sin^4 \frac{2\pi}{3} + 0 + 2 \sin^4 \frac{4\pi}{3} + 4 \sin^4 \frac{5\pi}{3} + 0 \right).$$

$$\int_0^\pi \sin^4(x) dx \approx \frac{3\pi}{4}. \quad (1,5) + (0,5)$$

2.

$$\int \frac{\ln(x) + 1}{1 + (x \ln x)^2} dx \stackrel{u=x \ln x}{=} \int \frac{du}{1 + u^2} \quad (2)$$

$$= \tan^{-1}(x \ln x) + c. \quad (1)$$

3.

$$\int \frac{x + \sin^{-1} x}{\sqrt{1 - x^2}} dx = \int \frac{x}{\sqrt{1 - x^2}} dx + \int \frac{\sin^{-1} x}{\sqrt{1 - x^2}} dx$$

$$= -\sqrt{1 - x^2} + \frac{1}{2}(\sin^{-1} x)^2 + c$$

(1.5) (1.5)

4.

$$\int \frac{x^2}{\sqrt{1 + x^6}} dx = \frac{1}{3} \int \frac{3x^2}{\sqrt{1 + x^6}} dx \quad (0.5)$$

$$= \frac{1}{3} \sinh^{-1}(x^3) + c \quad (1.5)$$

5.

$$\int \frac{dx}{\sqrt{4 - e^x}} \stackrel{u^2=e^x}{=} 2 \int \frac{du}{u\sqrt{4 - u^2}} \quad (2)$$

$$= -\operatorname{sech}^{-1}\left(\frac{e^{\frac{x}{2}}}{2}\right) \quad (1)$$

6.

$$\begin{aligned} \int \sin^5 x \cos^2 x dx &\stackrel{u=\cos x}{=} - \int (1-u^2)^2 u^2 du \quad (2) \\ &= -\frac{1}{3} \cos^3 x - \frac{1}{7} \cos^7 x + \frac{2}{5} \cos^5 x + c \quad (1) \end{aligned}$$

$$7. Y = \left(1 + \frac{7}{x}\right)^{5x}, \ln Y = \frac{\ln\left(1 + \frac{7}{x}\right)}{\frac{1}{5x}} \xrightarrow{x \rightarrow +\infty} 35. \quad (2.5)$$

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{7}{x}\right)^{5x} = e^{35}. \quad (0.5)$$

8.

$$\begin{aligned} \int x^2 \log_3 x dx &= \frac{1}{\ln 3} \int x^2 \ln x dx = \frac{1}{\ln 3} \left(\frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx \right) \quad (2) \\ &= \frac{1}{3 \ln 3} \left(x^3 \ln x - \frac{x^3}{3} \right) + c \quad (1) \end{aligned}$$

9.

$$\begin{aligned} \int \frac{x^2}{\sqrt{3-x^2}} dx &\stackrel{x=\sqrt{3}\sin\theta}{=} 3 \int \sin^2 \theta d\theta \\ &= \frac{3}{2}(\theta - \sin \theta \cos \theta) + c \quad (2) \\ &= \frac{3}{2} \left(\sin^{-1} \left(\frac{x}{\sqrt{3}} \right) - \frac{x\sqrt{3-x^2}}{3} \right) + c \quad (1) \end{aligned}$$

$$10. y = (\cos x)^x, \frac{y'}{y} = \ln \cos x - \frac{x \sin x}{\cos x} \quad (1.5)$$

$$y' = \left(\ln \cos x - \frac{x \sin x}{\cos x} \right) y \quad (0.5)$$

$$11. \frac{7x + 10}{(x + 1)(x^2 - 4)} = -\frac{1}{x + 1} + \frac{2}{x - 2} - \frac{1}{x + 2} \quad (2)$$
$$\int \frac{7x + 10}{(x + 1)(x^2 - 4)} dx = -\ln |x + 1| + 2 \ln |x - 2| - \ln |x + 2| + c$$

(1)