

1. $S_4 = \frac{3-1}{12} (f(1) + 4f(\frac{3}{2}) + 2f(2) + 4f(\frac{5}{2}) + f(3)) \approx 4.505$ (1.5) + (0.5)

2.

$$\int \frac{(1 - \frac{1}{x^2})^5}{x^3} dx \stackrel{t=1-\frac{1}{x^2}}{=} \frac{1}{2} \int t^5 dt \quad (2)$$

$$= \frac{1}{12} (1 - \frac{1}{x^2})^6 + c. \quad (1)$$

3. $\ln(y) = \frac{1}{2} \ln x + \frac{1}{3} \ln(x+2) + \frac{1}{5} \ln(x-1).$ (1.5)

Then $\frac{y'}{y} = \frac{1}{2x} + \frac{1}{3(x+2)} + \frac{1}{5(x-1)}.$

So $y' = \left(\frac{1}{2x} + \frac{1}{3(x+2)} + \frac{1}{5(x-1)} \right) \sqrt{x} \cdot \sqrt[3]{x+2} \cdot \sqrt[5]{x-1}.$ (0.5)

4.

$$\int \frac{(\sec x)^2}{\sqrt{4 - (\tan x)^2}} dx \stackrel{t=\tan x}{=} \int \frac{dt}{\sqrt{4 - t^2}} \quad (1)$$

$$= \sin^{-1}\left(\frac{t}{2}\right) + c = \sin^{-1}\left(\frac{\tan x}{2}\right) + c. \quad (1)$$

5.

$$\int \frac{dx}{\sqrt{e^{2x} - 1}} \stackrel{t=e^x}{=} \int \frac{dt}{t\sqrt{t^2 - 1}} \quad (2)$$

$$= \sec^{-1}(e^x) + c. \quad (1)$$

6.

$$\int \frac{dx}{x\sqrt{1 - x^5}} \stackrel{t^2=x^5}{=} \frac{2}{5} \int \frac{dt}{t\sqrt{1 - t^2}} \quad (2)$$

$$= -\frac{2}{5} \operatorname{sech}^{-1}(x^{\frac{5}{2}}) + c \quad (1)$$

7.

$$\lim_{x \rightarrow 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4} = \lim_{x \rightarrow 0} \frac{-\sin x + x}{4x^3} \quad (1)$$

$$= \lim_{x \rightarrow 0} \frac{-\cos x + 1}{12x^2} \quad (1)$$

$$= \lim_{x \rightarrow 0} \frac{\sin x}{24x} = \frac{1}{24}. \quad (1)$$

8.

$$\begin{aligned} \int (\ln x)^2 dx &\stackrel{u=(\ln x)^2, v'=1}{=} x(\ln x)^2 - 2 \int \ln x dx \quad (1.5) \\ &= x(\ln x)^2 - 2x \ln x + 2x + c. \quad (1.5) \end{aligned}$$

9.

$$\begin{aligned} \int (\tan x)^5 (\sec x)^3 dx &\stackrel{t=\sec x}{=} \int (t^2 - 1)^2 t^2 dt \quad (1) \\ &= \frac{1}{7} \sec^7 x - \frac{2}{5} \sec^5 x + \frac{1}{3} \sec^3 x + c. \quad (2) \end{aligned}$$

10.

$$\begin{aligned} \int \frac{x^2}{\sqrt{9-x^2}} dx &\stackrel{x=3\sin\theta}{=} 9 \int \sin^2 \theta d\theta = \frac{9}{2} \int (1 - \cos(2\theta)) d\theta \quad (1) \\ &= \frac{9}{2} (\theta - \sin \theta \cos \theta) + c \quad (0.5) \\ &= \frac{9}{2} \left(\sin^{-1} \left(\frac{x}{3} \right) - \frac{1}{9} x \sqrt{9-x^2} \right) + c \quad (1.5) \end{aligned}$$

11.
$$\frac{x^2 + 8x + 10}{-x^2 - 6x - 11} \Big| \frac{x^2 + 6x + 11}{1}$$

$$2x - 1$$

$$\begin{aligned} \int \frac{x^2 + 8x + 10}{x^2 + 6x + 11} dx &= \int 1 + \frac{2x - 1}{x^2 + 6x + 11} dx \quad (1) \\ &= \int 1 + \frac{2x + 6}{x^2 + 6x + 11} dx - \int \frac{7}{(x+3)^2 + 2} dx \\ &= x + \ln(x^2 + 6x + 11) - \frac{7}{\sqrt{2}} \tan^{-1} \frac{(x+3)}{\sqrt{2}} + c. \quad (2) \end{aligned}$$