# Homework Assignment 1: SOLUTIONS

## Exercise 1

Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$ and $B = \{-4, -2, 0, 2, 4\}$ be subsets of the Universal Set $U = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$. Verify by calculation that: $(A \cup B^c)^c = A^c \cap B$.

**1.** Let A = { –3, –2, –1, 0, 1, 2, 3} and B = {–4, –2, 0, 2, 4} be subsets of the Universal Set U = {–5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5}. Verify by calculation that: $(A \cup B^c)^c = A^c \cap B$ .

$B^c = U - B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\} - \{-4, -2, 0, 2, 4\} = \{-5, -3, -1, 1, 3, 5\}$, so

$A \cup B^c = \{-3, -2, -1, 0, 1, 2, 3\} \cup \{-5, -3, -1, 1, 3, 5\} = \{-5, -3, -2, -1, 0, 1, 2, 3, 5\}$, thus

$(A \cup B^c)^c = U - (A \cup B^c) = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\} - \{-5, -3, -2, -1, 0, 1, 2, 3, 5\}$
$= \{-4, 4\}$.

$A^c = U - A = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\} - \{-3, -2, -1, 0, 1, 2, 3\} = \{-5, -4, 4, 5\}$,

hence $A^c \cap B = \{-5, -4, 4, 5\} \cap \{-4, -2, 0, 2, 4\} = \{-4, 4\}$.

Therefore, $(A \cup B^c)^c = \{-4, 4\} = A^c \cap B$.

## Exercise 2

If $A = \{\{1\}, \emptyset\}$, find the Power Set of $A$.

**2.** If A = { {1} , ∅ }, find the Power Set of A.

**P**(A) = { ∅, { {1} }, { ∅ }, { {1}, ∅ }  }

## Exercise 3

Find the truth table of the statement $[p \vee (q \wedge \neg r)] \rightarrow \neg q$.

| *p q r* | [ *p* | ∨ | ( *q* | ∧ | ~*r*)] | → | ~*q* |
|---|---|---|---|---|---|---|---|
| T T T | T | T | T | F | F | ***F*** | F |
| T T F | T | T | T | T | T | ***F*** | F |
| T F T | T | T | F | F | F | ***T*** | T |
| T F F | T | T | F | F | T | ***T*** | T |
| F T T | F | F | T | F | F | ***T*** | F |
| F T F | F | T | T | T | T | ***F*** | F |
| F F T | F | F | F | F | F | ***T*** | T |
| F F F | F | F | F | F | T | ***T*** | T |
| *Step* | *1* | *3* | *1* | *2* | *1* | *4* | *1* |

**Exercise 4**

Give the **converse**, **inverse**, **contrapositive** and **negation** of the statement: *All people who live in glass houses do not throw stones.*

Converse

All people who do not throw stones live in glass houses.

Inverse

All people who do not live in glass houses throw stones.

Contrapositive

All people who throw stones do not live in glass houses.

Negation

Some people live in glass houses and throw stones.

**Exercise 5**

For the given set of premises, show the following is a valid argument.

$$\neg r \wedge s$$
$$q \to r$$
$$p \wedge s \to t$$
$$p \vee q$$
$$\cdots\cdots\cdots\cdots$$
$$\therefore t$$

**Step1:** $\sim r \wedge s$

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$\therefore \sim r$

$\therefore s$

**Step 2:** $q \rightarrow r$

$\sim r$

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$\therefore \sim q$

**Step 3:** $p \vee q$

$\sim q$

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$\therefore p$

**Step 4:** $p$

$s$

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$\therefore p \wedge s$

**Step 5:** $p \wedge s \rightarrow t$

$p \wedge s$

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$\therefore t$

**Exercise 6**

Find a proposition with three variables $p$, $q$, and $r$ that is true when $p$ and $r$ are true and $q$ is false, and false otherwise.

Ans: (a) $p \wedge \neg q \wedge r$.

**Exercise 7**

Find a proposition with three variables $p$, $q$, and $r$ that is true when exactly one of the three variables is true, and false otherwise.

Ans: $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$.

**Exercise 8**

Find a proposition with three variables $p$, $q$, and $r$ that is never true.

Ans: $(p \wedge \neg p) \vee (q \wedge \neg q) \vee (r \wedge \neg r)$.

**Exercise 9**

Find a proposition using only $p$, $q$, $\neg$, and $\vee$ with the given truth table.

| $p$ | $q$ | ? |
|---|---|---|
| T | T | F |
| T | F | T |
| F | T | T |
| F | F | F |

Ans: $\neg(\neg p \vee q) \vee \neg(p \vee \neg q)$.

## Exercise 10

Determine whether $p \to (q \to r)$ and $p \to (q \wedge r)$ are equivalent.

Determine whether $p \to (q \to r)$ is equivalent to $(p \to q) \to r$.

Determine whether $(p \to q) \wedge (\neg p \to q) \equiv q$.

Write a proposition equivalent to $p \vee \neg q$ that uses only $p$, $q$, $\neg$ and the connective $\wedge$.

Write a proposition equivalent to $\neg p \wedge \neg q$ that uses only $p$, $q$, $\neg$ and the connective $\vee$.

12. Determine whether $p \to (q \to r)$ and $p \to (q \wedge r)$ are equivalent.
    Ans: Not equivalent. Let $q$ be false and $p$ and $r$ be true.

13. Determine whether $p \to (q \to r)$ is equivalent to $(p \to q) \to r$.
    Ans: Not equivalent. Let p, $q$, and $r$ be false.

14. Determine whether $(p \to q) \wedge (\neg p \to q) \equiv q$.
    Ans: Both truth tables are identical:

| $p$ | $q$ | $(p \to q) \wedge (\neg p \to q)$ | $q$ |
|---|---|---|---|
| T | T | T | T |
| T | F | F | F |
| F | T | T | T |
| F | F | F | F |
| | | | |

15. Write a proposition equivalent to $p \vee \neg q$ that uses only $p,q,\neg$ and the connective $\wedge$.
    Ans: $\neg(\neg p \wedge q)$.

16. Write a proposition equivalent to $\neg p \wedge \neg q$ using only $p,q,\neg$ and the connective $\vee$.
    Ans: $\neg(p \vee q)$.