



**TESTS FOR NOMINAL  
SCALEDATA:**

**CHI-SQUARE TEST**

# OBJECTIVE

In this lecture, you will learn the following items:

- How to perform a chi-square goodness-of-fit test.
- How to perform a chi-square test for independence.

# INTRODUCTION

Sometimes, data are best collected or conveyed nominally or categorically. These data are represented by counting the number of times a particular event or condition occurs.

In such cases, you may be seeking to determine if a given set of counts, or frequencies, statistically matches some known, or expected, set. Or, you may wish to determine if two or more categories are statistically independent.

In either case, we can use a nonparametric procedure to analyze nominal data.

In this lecture, we present three procedures for examining nominal data: chi square ( $\chi^2$ ) goodness of fit, 2-test for independence, and the Fisher exact test.

# THE $\chi^2$ GOODNESS-OF-FIT TEST

Some situations in research involve investigations and questions about relative frequencies and proportions for a distribution.

Some examples might include a comparison of the number of women pediatricians with the number of men pediatricians, a search for significant changes in the proportion of students entering a writing contest over 5 years, or an analysis of customer preference of three candy bar choices.

Each of these examples asks a question about a proportion in the population.

# Computing the $\chi^2$ Goodness-of-Fit Test Statistic

The  $\chi^2$  goodness-of-fit test is used to determine how well the obtained sample proportions or frequencies for a distribution fit the population proportions or frequencies specified in the null hypothesis.

The  $\chi^2$  statistic can be used when two or more categories are involved in the comparison. Formula 1 is referred to as Pearson's  $\chi^2$  and is used to determine the  $\chi^2$  statistic:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} \quad (1)$$

where  $f_o$  is the observed frequency (the data) and  $f_e$  is the expected frequency (the hypothesis).

Use Formula 2 to determine the expected frequency  $f_e$ :

$$f_e = P_i n \quad (2)$$

where  $P_i$  is a category's frequency proportion with respect to the other categories and  $n$  is the sample size of all categories and  $\sum f_o = n$ .

Use Formula 3 to determine the degrees of freedom for the  $\chi^2$  -test:

$$df = C - 1 \quad (3)$$

where  $C$  is the number of categories.

## Example

### $\chi^2$ Goodness-of-Fit Test (Category Frequencies Equal)

A marketing firm is conducting a study to determine if there is a significant preference for a type of chicken that is served as a fast food. The target group is college students. It is assumed that there is no preference when the study is started. The types of food that are being compared are chicken sandwich, chicken strips, chicken nuggets, and chicken taco.

The sample size for this study was  $n = 60$ . The data in Table 1 represent the observed frequencies for the 60 participants who were surveyed at the fast food restaurants.

**TABLE 1**

Chicken sandwich	Chicken strips	Chicken nuggets	Chicken taco
10	25	18	7

We want to determine if there is any preference for one of the four chicken fast foods that were purchased to eat by the college students. Since the data only need to be classified into categories, and no sample mean nor sum of squares needs to be calculated, the  $\chi^2$  statistic goodness-of-fit test can be used to test the nonparametric data.



# 1. State the Null and Research Hypotheses

The null hypothesis states that there is no preference among the different categories. There is an equal proportion or frequency of participants selecting each type of fast food that uses chicken.

The research hypothesis states that one or more of the chicken fast foods is preferred over the others by the college student.

The null hypothesis is:

$H_0$ : In the population of college students, there is no preference of one chicken fast food over any other.

$H_A$ : In the population of college students, there is at least one chicken fast food preferred over the others.

Thus, the four fast food types are selected equally often and the population distribution has the proportions shown in Table 2.

**TABLE 2**

Chicken sandwich	Chicken strips	Chicken nuggets	Chicken taco
25%	25%	25%	25%

## **2. Set the Level of Risk (or the Level of Significance) Associated with the Null Hypothesis**

The level of risk, also called an alpha ( $\alpha$ ), is frequently set at 0.05. We will use  $\alpha = 0.05$  in our example. In other words, there is a 95% chance that any observed statistical difference will be real and not due to chance.

### 3. Choose the Appropriate Test Statistic

The data are obtained from 60 college students who eat fast food chicken. Each student was asked which of the four chicken types of food he or she purchased to eat and the result was tallied under the corresponding category type.

The final data consisted of frequencies for each of the four types of chicken fast foods. These categorical data, which are represented by frequencies or proportions, are analyzed using the  $\chi^2$  goodness-of-fit test.

## 4. Compute the Test Statistic

First, tally the observed frequencies,  $f_o$ , for the 60 students who were in the study. Use these data to create the observed frequency table shown in Table 3.

**TABLE 3**

	Chicken sandwich	Chicken strips	Chicken nuggets	Chicken taco
Observed frequencies	10	25	18	7

Next, calculate the expected frequency for each category. In this case, the expected frequency,  $f_e$ , will be the same for all four categories since our research problem assumes that all categories are equal:

$$f_e = P_i n = \frac{1}{4}(60)$$

$$f_e = 15$$

Table 4 presents the expected frequencies for each category.

**TABLE 4**

	Chicken sandwich	Chicken strips	Chicken nuggets	Chicken taco
Expected frequencies	15	15	15	15

Using the values for the observed and expected frequencies, the  $\chi^2$  statistic may be calculated:

$$\begin{aligned}\chi^2 &= \sum \frac{(f_o - f_e)^2}{f_e} \\ &= \frac{(10-15)^2}{15} + \frac{(25-15)^2}{15} + \frac{(18-15)^2}{15} + \frac{(7-15)^2}{15} \\ &= \frac{(-5)^2}{15} + \frac{(10)^2}{15} + \frac{(3)^2}{15} + \frac{(-8)^2}{15} \\ &= \frac{25}{15} + \frac{100}{15} + \frac{9}{15} + \frac{64}{15} \\ &= 1.67 + 6.67 + 0.60 + 4.27 \\ \chi^2 &= 13.21\end{aligned}$$

## **5. Determine the Value Needed for Rejection of the Null Hypothesis Using the Appropriate Table of Critical Values for the Particular Statistic**

Before we go to the table of critical values, we must determine the degrees of freedom,  $df$ . In this example, there are four categories,  $C = 4$ .

To find the degrees of freedom, use  $df = C - 1 = 4 - 1$ . Therefore,  $df = 3$ .



Now, we use Table B.2, which lists the critical values for the  $\chi^2$ . The critical value is found in the 2 table for three degrees of freedom,  $df = 3$ .

Since we set the  $\alpha = 0.05$ , the critical value is 7.81.

A calculated value that is greater than or equal to 7.81 will lead us to reject the null hypothesis.

## 6. Compare the Obtained Value with the Critical Value

The critical value for rejecting the null hypothesis is 7.81 and the obtained value is  $\chi^2 = 13.21$ .

If the critical value is less than or equal to the obtained value, we must reject the null hypothesis. If instead, the critical value exceeds the obtained value, we do not reject the null hypothesis. Since the critical value is less than our obtained value, we must reject the null hypothesis.

Note that the critical value for  $\alpha = 0.01$  is 11.34. Since the obtained value is 13.21, a value greater than 11.34, the data indicate that the results are highly significant.

## 7. Interpret the Results

We rejected the null hypothesis, suggesting that there is a real difference among chicken fast food choices preferred by college students.

In particular, the data show that a larger portion of the students preferred the chicken strips and only a few of them preferred the chicken taco.

## 8. Reporting the Results

The reporting of results for the  $\chi^2$  goodness of fit should include such information as the total number of participants in the sample and the number that were classified in each category. In some cases, bar graphs are

good methods of presenting the data. In addition, include the  $\chi^2$  statistic, degrees of freedom, and p-value's relation to  $\alpha$ . For this study, the number of students who ate each type of chicken fast food should be noted either in a table or plotted on a bar graph. The probability,  $p < 0.01$ , should also be indicated with the data to show the degree of significance of the  $\chi^2$ .

For this example, 60 college students were surveyed to determine which fast food type of chicken they purchased to eat. The four choices were chicken sandwich, chicken strips, chicken nuggets, and chicken taco. Student choices were 10, 25, 18, and 7, respectively.

The  $\chi^2$  goodness-of-fit test was significant:

$$(\chi^2_{(3)} = 13.21, p < 0.01).$$

Based on these results, a larger portion of the students preferred the chicken strips while only a few students preferred the chicken taco.

## Example

### $\chi^2$ Goodness-of-Fit Test (Category Frequencies Not Equal)

Sometimes, research is being conducted in an area where there is a basis for different expected frequencies in each category. In this case, the null hypothesis will indicate different frequencies for each of the categories according to the expected values.

These values are usually obtained from previous data that were collected in similar studies.

In this study, a school system uses three different physical fitness programs because of scheduling requirements.

A researcher is studying the effect of the programs on 10th-grade students' 1-mile run performance.

Three different physical fitness programs were used by the school system and will be described later.

- *Program 1.* Delivers health and physical education in 9-week segments with an alternating rotation of nine straight weeks of health education and then nine straight weeks of physical education.
- *Program 2.* Delivers health and physical education everyday with 30 min for health, 10 min for dress-out time, and 50 min of actual physical activity
- *Program 3.* Delivers health and physical education in 1-week segments with an alternating rotation of 1 week of health education and then 1 week of physical education

Using students who participated in all three programs, the researcher is comparing these programs based on student performances on the 1-mile run. The researcher recorded the program in which each student received the most benefit. Two hundred fifty students had participated in all three programs. The results for all of the students are recorded in Table 5.



**TABLE 5**

Program 1	Program 2	Program 3
110	55	85

We want to determine if the frequency distribution in the case earlier is different from previous studies. Since the data only need to be classified into categories, and no sample mean or sum of squares needs to be calculated, the  $\chi^2$  goodness-of-fit test can be used to test the nonparametric data.

# 1. State the Null and Research Hypotheses

The null hypothesis states the proportion of students who benefited most from one of the three programs based on a previous study. As shown in Table 8.6, there are unequal expected frequencies for the null hypothesis. The research hypothesis states that there is at least one of the three categories that will have a different proportion or frequency than those identified in the null hypothesis.

The hypothesis is:

$H_0$ : The proportions do not differ from the previously determined proportions shown in Table 6.

$H_A$ : The population distribution has a different shape than that specified in the null hypothesis.

**TABLE 6**

Program 1	Program 2	Program 3
32%	22%	45%

## **2. Set the Level of Risk (or the Level of Significance) Associated with the Null Hypothesis**

The level of risk, also called an alpha ( $\alpha$ ), is frequently set at 0.05. We will use  $\alpha = 0.05$  in our example. In other words, there is a 95% chance that any observed statistical difference will be real and not due to chance.

### 3. Choose the Appropriate Test Statistic

The data are being obtained from the 1-mile run performance of 250 10th-grade students who participated in a school system's three health and physical education programs. Each student was categorized based on the plan in which he or she benefited most.

The final data consisted of frequencies for each of the three plans. These categorical data which are represented by frequencies or proportions are analyzed using the  $\chi^2$  goodness-of-fit test.

## Compute the Test Statistic

First, tally the observed frequencies for the 250 students who were in the study. This was performed by the researcher. Use the data to create the observed frequency table shown in Table 7.

**TABLE 7**

	Program 1	Program 2	Program 3
Observed frequencies	110	55	85

Next, calculate the expected frequencies for each category. In this case, the expected frequency will be different for each category. Each one will be based on proportions stated in the null hypothesis:

$$f_{ei} = P_i n$$

$$f_e \text{ for program 1} = 0.32(250) = 80$$

$$f_e \text{ for program 2} = 0.22(250) = 55$$

$$f_e \text{ for program 3} = 0.46(250) = 115$$

Table 8 presents the expected frequencies for each category.

**TABLE 8**

	Program 1	Program 2	Program 3
Expected frequencies	80	55	115



Use the values for the observed and expected frequencies calculated earlier to calculate the  $\chi^2$  statistic:

$$\begin{aligned}\chi^2 &= \sum \frac{(f_o - f_e)^2}{f_e} \\ &= \frac{(110 - 80)^2}{80} + \frac{(55 - 55)^2}{55} + \frac{(85 - 115)^2}{115} \\ &= \frac{30^2}{80} + \frac{0^2}{55} + \frac{-30^2}{115} \\ &= 11.25 + 0 + 7.83 \\ \chi^2 &= 19.08\end{aligned}$$

## 5. Determine the Value Needed for Rejection of the Null Hypothesis

Using the Appropriate Table of Critical Values for the Particular Statistic. Before we go to the table of critical values, we must determine the degrees of freedom,  $df$ . In this example, there are four categories,  $C = 3$ .

To find the degrees of freedom, use  $df = C - 1 = 3 - 1$ . Therefore,  $df = 2$ .

Now, we use Table B.2, which lists the critical values for the  $\chi^2$ . The critical value is found in the 2 table for two degrees of freedom,  $df = 2$ .

Since we set  $\alpha = 0.05$ , the critical value is 5.99. A calculated value that is greater than 5.99 will lead us to **reject the null hypothesis.**

## 6. Compare the Obtained Value with the Critical Value

The critical value for rejecting the null hypothesis is 5.99 and the obtained value is  $\chi^2 = 19.08$ .

If the critical value is less than or equal to the obtained value, we must reject the null hypothesis. If instead, the critical value exceeds the obtained value, we do not reject the null hypothesis. Since the critical value is less than our obtained value, we must reject the null hypothesis.

Note that the critical value for  $\alpha = 0.01$  is 9.21. Since the obtained value is 19.08, which is greater than the critical value, the data indicate that the results are highly significant.

## 7. Interpret the Results

We rejected the null hypothesis, suggesting that there is a real difference in how the health and physical education program affects the performance of students on the 1-mile run as compared with the existing research.

By comparing the expected frequencies of the past study and those obtained in the current study, it can be noted that the results from program 2 did not change. Program 2 was least effective in both cases, with no difference between the two. Program 1 became more effective and program 3 became less effective.

## 8. Reporting the Results

The reporting of results for the  $\chi^2$  goodness of fit should include such information as the total number of participants in the sample, the number that were classified in each category, and the expected frequencies that are being used for comparison. It is important also to cite a source for the expected frequencies so that the decisions made from the study can be supported.

In addition, include the  $\chi^2$  statistic, degrees of freedom, and p-value's relation to  $\alpha$ .

It is often a good idea to present a bar graph to display the observed and expected frequencies from the study. For this study, the probability,  $p < 0.01$ , should also be indicated with the data to show the degree of significance of the  $\chi^2$ .

For this example, 250 10th-grade students participated in three different health and physical education programs.

Using 1-mile run performance, students' program of greatest benefit was compared with the results from past research. The  $\chi^2$  goodness-of-fit test was significant ( $\chi^2_{(2)}=19.08, p < 0.01$ ).

Based on these results, program 2 was least effective in both cases, with no difference between the two.

Program 1 became more effective and program 3 became less effective.

# THE $\chi^2$ TEST FOR INDEPENDENCE

Some research involves investigations of frequencies of statistical associations of two categorical attributes. Examples might include a sample of men and women who bought a pair of shoes or a shirt. The first attribute,  $A$ , is the gender of the shopper with two possible categories:

men =  $A_1$

women =  $A_2$



The second attribute,  $B$ , is the clothing type purchased by each individual:

pair of shoes =  $B_1$

shirt =  $B_2$

We will assume that each person purchased only one item, either a pair of shoes or a shirt. The entire set of data is then arranged into a joint-frequency distribution table.

Each individual is classified into one category which is identified by a pair of categorical attributes (see Table 9).

**TABLE 9**

	$A_1$	$A_2$
$B_1$	$(A_1, B_1)$	$(A_2, B_1)$
$B_2$	$(A_1, B_2)$	$(A_2, B_2)$

The  $\chi^2$  -test for independence uses sample data to test the hypothesis that there is no statistical association between two categories.

In this case, whether there is a significant association between the gender of the purchaser and the type of clothing purchased. The test determines how well the sample proportions fit the proportions specified in the null hypothesis.

# 1. Computing the $\chi^2$ Test for Independence

The  $\chi^2$  -test for independence is used to determine whether there is a statistical association between two categorical attributes. The  $\chi^2$  statistic can be used when two or more categories are involved for two attributes.

Formula 4 is referred to as Pearson's  $\chi^2$  and is used to determine the  $\chi^2$  statistic:

$$\chi^2 = \sum_j \sum_k \frac{(f_{ojk} - f_{ejk})^2}{f_{ejk}} \quad (4)$$

where  $f_{ojk}$  is the observed frequency for cell  $A_j B_k$  and  $f_{ejk}$  is the expected frequency for cell  $A_j B_k$ .

In tests for independence, the expected frequency  $f_{ejk}$  in any cell is found by multiplying the row total times the column total and dividing the product by the grand total  $N$ . Use Formula 5 to determine the expected frequency  $f_{ejk}$ :

$$f_{ejk} = \frac{(\text{freq. } A_j)(\text{freq. } B_k)}{N} \quad (5)$$

The degrees of freedom  $df$  for the  $\chi^2$  is found using Formula 6:

$$df = (R - 1)(C - 1) \quad (6)$$

where  $R$  is the number of rows and  $C$  is the number of columns.

It is important to note that Pearson's  $\chi^2$  formula returns a value that is too small when data form a  $2 \times 2$  contingency table. This increases the chance of a type I error.

In such a circumstance, one might use the Yates's continuity correction shown in Formula 7:

$$\chi^2 = \sum_j \sum_k \frac{(f_{ojk} - f_{ejk} - 0.5)^2}{f_{ejk}} \quad (7)$$

Daniel (1990) has cited a number of criticisms to the Yates's continuity correction.

While he recognizes that the procedure has been frequently used, he also observes a decline in its popularity.

Toward the end of this course, we present an alternative for analyzing a  $2 \times 2$  contingency table using the **Fisher exact test**.

At this point, the analysis is limited to identifying an association's presence or absence. In other words, the  $\chi^2$  -test's level of significance does not describe the strength of its association.

We can use the effect size to analyze the degree of association.

For the 2-test for independence, the effect size between the nominal variables of a  $2 \times 2$  contingency table can be calculated and represented with the phi ( $\phi$ ) coefficient (see Formula 8):

$$\phi = \sqrt{\frac{\chi^2}{n}} \quad (8)$$

Where  $\chi^2$  is the chi-square test statistic and  $n$  is the number in the entire sample.



The  $\phi$  coefficient ranges from 0 to 1. Cohen (1988) defined the conventions for effect size as small = 0.10, medium = 0.30, and large = 0.50. (Correlation coefficient and effect size are both measures of association.)

When the 2 contingency table is larger than  $2 \times 2$ , Cramer's  $V$  statistic may be used to express effect size. The formula for Cramer's  $V$  is shown in Formula 9:

$$V = \sqrt{\frac{\chi^2}{(n)(L - 1)}} \quad (9)$$

where  $\chi^2$  is the chi-square test statistic,  $n$  is the total number in the sample, and  $L$  is the minimum value of the row totals and column totals from the contingency table.

## Example

### $\chi^2$ Test for Independence

A counseling department for a school system is conducting a study to investigate the association between children's attendance in public and private preschool and their behavior in the kindergarten classroom.

It is the researcher's desire to see if there is any positive association between early exposure to learning and behavior in the classroom.

The sample size for this study was  $n = 100$ . The following data in Table 10 represent the observed frequencies for the 100 children whose behavior as observed during their first 6 weeks of school. The students who were in the study were identified by the type of preschool educational exposure they received.

**TABLE 10**

	Behavior in kindergarten			Row totals
	Poor	Average	Good	
Public preschool	12	25	10	47
Private preschool	6	12	0	18
No preschool	2	23	10	35
Column totals	20	60	20	100

We want to determine if there is any association between type of preschool experience and behavior in kindergarten in the first 6 weeks of school. Since the data only need to be classified into categories, and no sample mean nor sum of squares needs to be calculated, the  $\chi^2$  statistic for independence can be used to test the nonparametric data.

# 1. State the Null and Research Hypotheses

The null hypothesis states that there is no association between the two categories. The behavior of the children in kindergarten is independent of the type of preschool experience they had. The research hypothesis states that there is a significant association between the preschool experience of the children and their behavior in kindergarten.

The null hypothesis is

$H_0$ : In the general population, there is no association between type of preschool experience a child has and the child's behavior in kindergarten.

The research hypothesis states

$H_A$ : In the general population, there is a predictable relationship between the preschool experience and the child's behavior in kindergarten.

## **2. Set the Level of Risk (or the Level of Significance) Associated with the Null Hypothesis**

The level of risk, also called an alpha ( $\alpha$ ), is frequently set at 0.05. We will use  $\alpha = 0.05$  in our example. In other words, there is a 95% chance that any observed statistical difference will be real and not due to chance.

### 3. Choose the Appropriate Test Statistic

The data are obtained from 100 children in kindergarten who experienced differing preschool preparation prior to entering formal education. The kindergarten teachers for the children were asked to rate students' behaviors using three broad levels of ratings obtained from a survey.

The students were then divided up into three groups according to preschool experience (no preschool, private preschool, and public preschool). These data are organized into a two-dimensional categorical distribution that can be analyzed using an independent  $\chi^2$ -test.

## 4. Compute the Test Statistic

First, tally the observed frequencies  $f_{ojk}$  for the 100 students who were in the study.

Use these data to create the observed frequency table shown in Table 11.

**TABLE 11**

	Behavior in kindergarten (observed)			Row totals
	Poor	Average	Good	
Public preschool	12	25	10	47
Private preschool	6	12	0	18
No preschool	2	23	10	35
Column totals	20	60	20	100



Next, calculate the expected frequency  $f_{ejk}$  for each category:

$$f_{e11} = \frac{(47)(20)}{100} = 940/100 = 9.4$$

$$f_{e12} = \frac{(47)(60)}{100} = 2820/100 = 28.2$$

$$f_{e13} = \frac{(47)(20)}{100} = 940/100 = 9.4$$

$$f_{e21} = \frac{(18)(20)}{100} = 360/100 = 3.6$$

$$f_{e22} = \frac{(18)(60)}{100} = 1080/100 = 10.8$$

$$f_{e23} = \frac{(18)(20)}{100} = 360/100 = 3.6$$

$$f_{e31} = \frac{(35)(20)}{100} = 700/100 = 7.0$$

$$f_{e32} = \frac{(35)(60)}{100} = 2100/100 = 21.0$$

$$f_{e33} = \frac{(35)(20)}{100} = 700/100 = 7.0$$

Place these values in an expected frequency table (see Table 12).

**TABLE 12**

	Behavior in kindergarten (expected)			Row totals
	Poor	Average	Good	
Public preschool	9.4	28.2	9.4	47
Private preschool	3.6	10.8	3.6	18
No preschool	7.0	21.0	7.0	35
Column totals	20	60	20	100

Using the values for the observed and expected frequencies in Tables 11 and 12, the  $\chi^2$  statistic can be calculated:

$$\begin{aligned}
 \chi^2 &= \sum_j \sum_k \frac{(f_{ojk} - f_{ejk})^2}{f_{ejk}} \\
 &= \frac{(12 - 9.4)^2}{9.4} + \frac{(25 - 28.2)^2}{28.2} + \frac{(10 - 9.4)^2}{9.4} + \frac{(6 - 3.6)^2}{3.6} + \frac{(12 - 10.8)^2}{10.8} \\
 &\quad + \frac{(0 - 3.6)^2}{3.6} + \frac{(2 - 7)^2}{7} + \frac{(23 - 21)^2}{21} + \frac{(10 - 7)^2}{7} \\
 &= \frac{(2.6)^2}{9.4} + \frac{(-3.2)^2}{28.2} + \frac{(0.6)^2}{9.4} + \frac{(2.4)^2}{3.6} + \frac{(1.2)^2}{10.8} \\
 &\quad + \frac{(-3.6)^2}{3.6} + \frac{(-5)^2}{7} + \frac{(2)^2}{21} + \frac{(3)^2}{7} \\
 &= 0.72 + 0.36 + 0.04 + 1.60 + 0.13 + 3.60 + 3.57 + 0.19 + 1.29 \\
 \chi^2 &= 11.50
 \end{aligned}$$

## 5. Determine the Value Needed for Rejection of the Null Hypothesis

Using the Appropriate Table of Critical Values for the Particular Statistic.

Before we go to the table of critical values, we need to determine the degrees of freedom,  $df$ .

In this example, there are three categories in the preschool experience dimension,  $R = 3$ , and three categories in the behavior dimension,  $C = 3$ .

To find the degrees of freedom, use

$$df = (R - 1)(C - 1) = (3 - 1)(3 - 1).$$

Therefore,  $df = 4$ .

Now, we use Table B.2, which lists the critical values for the  $\chi^2$ .

The critical value is found in the  $\chi^2$  table for four degrees of freedom,  $df = 4$ .

Since we set  $\alpha = 0.05$ , the critical value is 9.49.

A calculated value that is greater than or equal to 9.49 will lead us to reject the null hypothesis.

## 6. Compare the Obtained Value with the Critical Value

The critical value for rejecting the null hypothesis is 9.49 and the obtained value is  $\chi^2 = 11.50$ .

If the critical value is less than or equal to the obtained value, we must reject the null hypothesis. If instead, the critical value exceeds the obtained value, we do not reject the null hypothesis. Since the critical value is less than our obtained value, we must reject the null hypothesis.

## 7. Interpret the Results

We rejected the null hypothesis, suggesting that there is a real association between type of preschool experience children obtained and their behavior in the kindergarten classroom during their first few weeks in school. In particular, data tend to show that children who have private schooling do not tend to get good behavior ratings in school.

The other area that tends to show some significant association is between poor behavior and no preschool experience.

The students who had no preschool had very few poor behavior ratings in comparison with the other two groups.

At this point, the analysis is limited to identifying an association's presence or absence.

In other words, the  $\chi^2$ -test's level of significance does not describe the strength of its association.

The American Psychological Association (2001), however, has called for a measure of the degree of association called the effect size.

For the  $\chi^2$ -test for independence with a  $3 \times 3$  contingency table, we determine the strength of association, or the effect size, using Cramer's V.



From Table 11, we find that  $L = 3$ . For  $n = 100$  and  $\chi^2 = 11.50$ , we use Formula 9 to determine  $V$ :

$$\begin{aligned} V &= \sqrt{\frac{11.50}{(100)(3-1)}} \\ &= \sqrt{0.06} \\ V &= 0.24 \end{aligned}$$

Our effect size, Cramer's  $V$ , is 0.24. This value indicates a medium level of association between type of preschool experience children obtained and their behavior in the kindergarten classroom during their first few weeks in school.

## 8. Reporting the Results

The reporting of results for the  $\chi^2$  -test for independence should include such information as the total number of participants in the sample and the number of participants classified in each of the categories.

In addition, include the  $\chi^2$  statistic, degrees of freedom, and the p-value's relation to .

For this study, the number of children who were in each category (including preschool experience and behavior rating) should be presented in the two-dimensional table (see Table 10).

For this example, the records of 100 kindergarten students were examined to determine whether there was an association between preschool experience and behavior in kindergarten.

The three preschool experiences were no preschool, private preschool, and public preschool. The three behavior ratings were poor, average, and good.

The 2-test was significant ( $\chi^2_{(4)} = 11.50, p < 0.05$ ).

Moreover, our effect size, using Cramer's V, was 0.24. Based on the results, there was a tendency shown for students with private preschool to not have good behavior and those with no preschool to not have poor behavior.

It also indicated that average behavior was strong for all three preschool experiences.

# SUMMARY

Nominal, or categorical, data sometimes need analyses. In such cases, you may be seeking to determine if the data statistically match some known or expected set of frequencies. Or, you may wish to determine if two or more categories are statistically independent. In either case, nominal data can be analyzed with a nonparametric procedure.

In this lecture, we presented three procedures for examining nominal data:

chi-square ( $\chi^2$ ) goodness of fit and  $\chi^2$  -test for independence.