Lecture 16

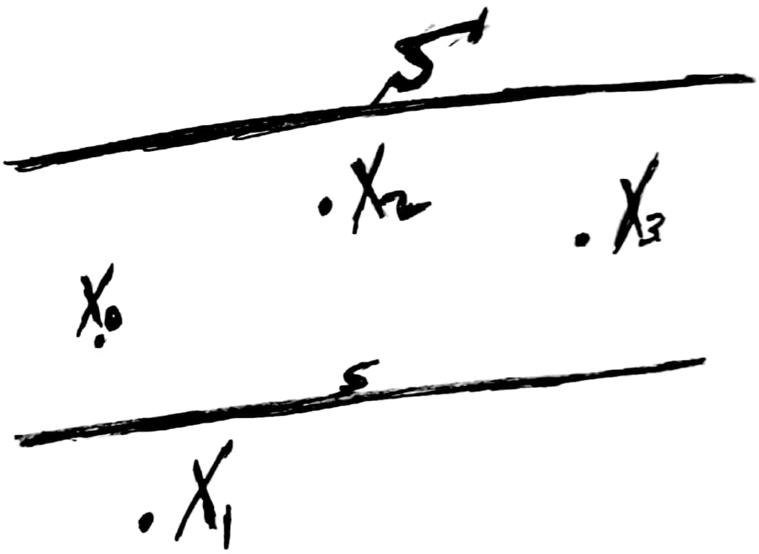
1// Sec. 3.3 Some Markov chain Models

3.3.1 An Inventory Model   p 87

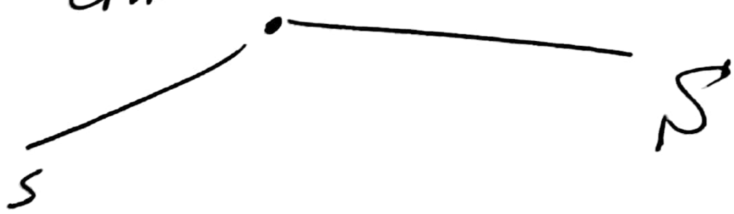
كمية المخزون X_n المخزون stock value at the end of an interval n perior to replenishment.

الطلب ξ_n total demand for the Commodity السلعة

N.B.



* Replenishment of stock depends on 2 levels (two non-negative critical nos)



* stock values $X_0, X_1, X_2, \ldots$ Constitute a Markov chain

2

* The states of the process $\{X_n\}$ are

عدم تجديد   تحتاج إلى تجديد

$$\underbrace{S', S'-1, \ldots, +1}, \underbrace{0, -1, \ldots}$$

s

Not that: If $X_n \leq s \Longrightarrow$ replenishment

* the transition probs given by

$$P_{ij} = pr\{X_{n+1} = j \mid X_n = i\}$$

$$= \begin{cases} pr(\xi_{n+1} = S' - j) & \text{if } i \leq s \quad \text{تجديد} \\ pr(\xi_{n+1} = i - j) & \text{if } s < i \leq S' \quad \text{عدم التجديد} \end{cases}$$

EX p. 89 Textbook

Consider a spare parts inventory model as a numerical example in which either 0, 1, or 2 repair parts are demanded in any period, with $pr\{\xi_n = 0\} = 0.5$, $pr\{\xi_n = 1\} = 0.4$ and $pr\{\xi_n = 2\} = 0.1$

and suppose $s = 0$, while $S' = 2$. Determine the transition prob. Mx

Ans

$$P = \begin{array}{c c} & \begin{matrix} -1 & 0 & 1 & 2 \end{matrix} \\ \begin{matrix} \text{تجديد} \; -1 \\ 0 \\ \text{عدم تجديد} \; 1 \\ 2 \end{matrix} & \begin{bmatrix} 0 & 0.1 & 0.4 & 0.5 \\ 0 & 0.1 & 0.4 & 0.5 \\ 0.1 & 0.4 & 0.5 & 0 \\ 0 & 0.1 & 0.4 & 0.5 \end{bmatrix} \end{array},$$

$$P_{-1,-1} = pr\{X_{n+1} = -1 \mid X_n = -1\}$$
$$= pr(\xi_{n+1} = 2+1) = pr(\xi_{n+1} = 3) = 0$$

$$P_{0,2} = pr\{X_{n+1} = 2 \mid X_n = 0\}$$
$$= pr(\xi_{n+1} = 0) = 0$$

$$P_{1,0} = pr\{X_{n+1} = 0 \mid X_n = 1\}$$
$$= pr(\xi_{n+1} = 1) = 0.4$$

...

$$\ldots \; P_{2,2} = pr\{X_{n+1} = 2 \mid X_n = 2\}$$
$$= pr(\xi_{n+1} = 0) = 0.5$$

3 pb 3.3.1 p.93

Given $s=0$, $S=3$

the possible values for X_n are

3, 2, 1, 0 and -1 ~~states~~

no replenishment عدم تجدید — تجدید replenishment

$pr(\xi_n=0)=0.4$, $pr(\xi_n=1)=0.3$, $pr(\xi_n=2)=0.3$

Find the transition prob. $M\underline{X}$

Ans:

$$
\begin{array}{cc|c}
 & & \begin{matrix} -1 & \quad 0 & \quad 1 & \quad 2 & \quad 3 \end{matrix} \\
\text{تجدید} & \begin{matrix} -1 \\ 0 \end{matrix} & \\
\text{عدم تجدید} & \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} &
\end{array}
\begin{bmatrix}
0 & 0 & 0.3 & 0.3 & 0.4 \\
0 & 0 & 0.3 & 0.3 & 0.4 \\
0.3 & 0.3 & 0.4 & 0 & 0 \\
0 & 0.3 & 0.3 & 0.4 & 0 \\
0 & 0 & 0.3 & 0.3 & 0.4
\end{bmatrix}
$$

$P_{-1,-1} = pr(\xi_{n+1} = S-j)$, $i \leq s$

$= pr(\xi_{n+1} = 4) = 0$

$P_{0,-1} = pr(\xi_{n+1} = 4) = 0$

$P_{1,-1} = pr(\xi_{n+1} = i-j)$, $s < i \leq S$

$= pr(\xi_{n+1} = 2) = 0.3$

$P_{2,-1} = pr(\xi_{n+1} = 3) = 0$

$P_{3,-1} = pr(\xi_{n+1} = 4) = 0$

$P_{-1,0} = pr(\xi_{n+1} = S-j)$, $i \leq s$

$= pr(\xi_{n+1} = 3) = 0$

$P_{0,0} = pr(\xi_{n+1} = 3) = 0$

$P_{1,0} = pr(\xi_{n+1} = i-j)$, $s < i \leq S$

$= pr(\xi_{n+1} = 1) = 0.3$

$P_{2,0} = pr(\xi_{n+1} = 2) = 0.3$

$P_{3,0} = pr(\xi_{n+1} = 3) = 0$

$P_{3,3} = pr(\xi_{n+1} = 0) = 0.4$

$\#$