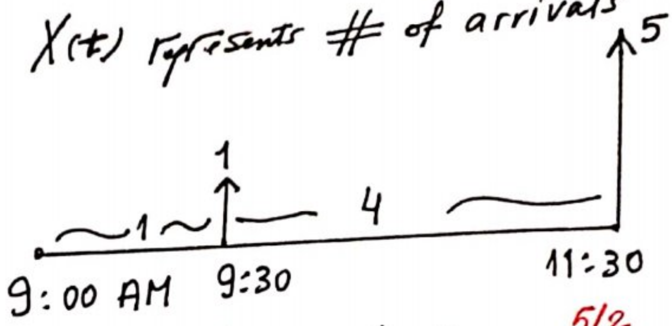
Poisson process تابع Lecture (26)

Ex (2) p. 226

In poisson process, $X(t)$ represents # of arrivals



معدل كلية بواسونه $\boxed{\lambda = 4}$ rate per hour

0 $\quad t = \frac{1}{2}$ $\quad \frac{1}{2}$ $\quad \lambda t = 4(\frac{1}{2}) = 2$

$t = 2$ $\quad \frac{5}{2}$ $\quad \lambda t = 4(2) = 8$

Note that: $X(\frac{1}{2}) - X(0)$, $X(\frac{5}{2}) - X(\frac{1}{2})$ are 2 independent r.v's

$pr\{X(\frac{1}{2}) = 1, X(\frac{5}{2}) = 5\}$?? اطلب

Ans:

$$pr\{X(\tfrac{1}{2}) = 1, X(\tfrac{5}{2}) = 5\}$$

$$= pr\{\underline{X(\tfrac{1}{2}) - X(0)} = 1, \underline{X(\tfrac{5}{2}) - X(\tfrac{1}{2})} = 4\}$$

2 indep. r.v's

$$= \frac{[4(\frac{1}{2})]^1 e^{-2}}{1!} \cdot \frac{[4(2)]^4 e^{-8}}{4!}$$

$$= 2e^{-2}\left(\frac{512}{3} e^{-8}\right)$$

$$\simeq \boxed{0.015}$$

$p(x, y) = p(x)\,p(y)$ for indep. r.v's X and Y

For poisson process

$$pr\{X(s+t) - X(s) = k\} = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0, 1, 2, \ldots$$

2//

- For non-homogeneous Poisson process

If $\lambda = \lambda(t)$

then $X(s+t) - X(s) \sim \text{poisson}(\mu)$

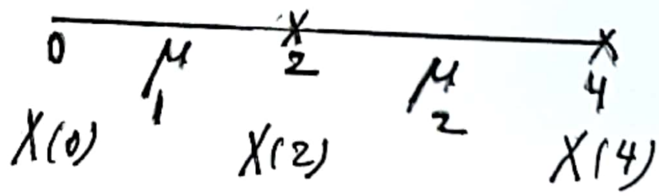
where $\mu = \int_{s}^{s+t} \lambda(u)\,du$

Ex p.227

$X(t)$ represents demands عدد الطلبات على مرفق الإسعاف في منطقة ما (الإسعافات الأولية)

معدل عملية بواسون

$$\lambda(t) = \begin{cases} 2t, & 0 \le t < 1 \\ 2, & 1 \le t < 2 \\ 4-t, & 2 \le t \le 4 \end{cases}$$



find

a) $pr\{X(2) = 2\}$

or find

a) prob. that two demands occur in the first 2h of operation

b) $pr\{X(4) - \underset{}{\overset{\text{المطلوب}}{X(2)}} = 2\}$

b) prob. that two demands occur in the second 2h

3

a) $\mu_1 = \int_0^2 \lambda(u)\,du$

$$= \int_0^1 2t\,dt + \int_1^2 2\,dt$$

$$= \left[\cancel{\tfrac{2t^2}{2}}\right]_0^1 + 2\left[t\right]_1^2$$

$$= 1 + 2(2-1) = 1 + 2 = 3$$

$pr\{X(2)=2\}$

$$= pr\{X(2) - X(0) = 2\}$$

$$= \frac{e^{-\mu_1}\mu_1^k}{k!} = \frac{e^{-3} \cdot 3^2}{2!} = \boxed{0.2240}$$

b) $\mu_2 = \int_2^4 \lambda(u)\,du = \int_2^4 (4-t)\,dt$

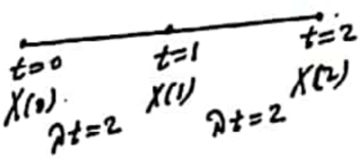
$$\mu_2 = \left[4t - \frac{t^2}{2}\right]_2^4 = 8 - 6 = 2$$

$$\therefore\ pr\{X(4) - X(2) = 2\} = \frac{e^{-\mu_2}\mu_2^k}{k!}$$

$$= \frac{e^{-2} \cdot 2^2}{2!}$$

$$= 0.2707$$

Hw pb 5.1.1 p. 228



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