Lecture 3

Some Important Discrete Distributions

*** Bernoulli distn** توزيع برنولي

$$X \sim Ber(p) \quad , \quad 0 < p < 1$$

Its mass prob. fn is

$$pr(X = x) = p^{x}(1-p)^{1-x}, \quad x \in \{0, 1\}$$

Moments: $E(X) = p$, $Var(X) = p(1-p) = pq$

$q = 1 - p$ الفشل ← ، النجاح ← p

*** Binomial distn** توزيع ذي الحدين

$$X \sim Bin(n, p), \quad n \in N \text{ and } 0 < p < 1$$

$$pr(X = x) = \binom{n}{x} p^{x}(1-p)^{n-x}$$

for $x = 0, 1, 2, \dots, n$

Moments: $E(X) = np$, $Var(X) = np(1-p) = npq$

Note: $\binom{n}{x} = \dfrac{n!}{x!(n-x)!}$

* 2// poisson distn توزيع بواسون

$$X \sim \text{poisson}(\lambda), \quad \lambda > 0$$

$$pr(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots$$

Moments: $E(X) = \lambda$, $Var(X) = \lambda$

المتوسط التباين

* Geometric distn

$X \sim Geom(p)$, $0 < p < 1$

iff its mass prob. fn is

$$pr(X=k) = p(1-p)^k, \quad k = 0, 1, 2, \ldots$$

Note: X counts # of failures prior to the first success

عدد مرات الفشل

قبل تحقيق أول نجاح

$$E(X) = \frac{1-p}{p}, \quad Var(X) = \frac{1-p}{p^2}$$

OR $pr(X=k) = p(1-p)^{k-1}, \quad k = 1, 2, \ldots$

$$E(X) = \frac{1}{p}, \quad Var(X) = \frac{1-p}{p^2}$$

- pb 1.3.1 p. 24

# of elements of this event (total heads is three)

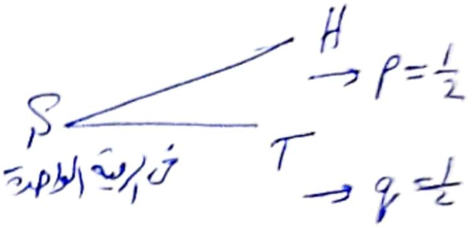
$$= C^5_3 = \binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} \cdot 2 \times 1} = 10$$

3

pr (that the total of heads is three)

$= pr\,(X=3)$

$= \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$



$= 10\left(\frac{1}{8}\right)\left(\frac{1}{4}\right) = \frac{10}{32} = \frac{5}{16}$

OK for tossing a coin 5 times

# of outcomes in the sample space is $2^5 = 32$

$\Rightarrow pr(X=3) = \frac{10}{32} = \frac{5}{10}$

Note: $C^5_3 = 10$ كما في الفقرة أ مثلاً

Pb 1.3.3 p.24

X count # of failures perior to the first success

$\Rightarrow X \sim \text{Geom}\,(p)$

$p = 0.05$

$pr(X=10) = p\,(1-p)^{k-1}, \quad k=1,2,\dots$

$= 0.05\,(1-0.05)^9$

$= 0.05\,(0.95)^9$

$= 0.0315$

إضافة يمكن حساب كذلك Variance (التباين) و mean (الوسط الحسابي)

$E(X) = 1/p, \quad Var(X) = \frac{1-p}{p^2}$

4

pb 1.2.9 p. 17 (Lecture ② Revise)

EX Determine the distribution function, mean & Variance corresponding to the triangular density.

$$f(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ 2-x & \text{for } 1 \leq x \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

Ans: $F_X(x) = \int_{-\infty}^{x} f(t)\,dt$

for $0 \leq x \leq 1$

$$F_X(x) = \int_0^x t\,dt$$

$$= \left[\frac{t^2}{2}\right]_0^x = \boxed{\frac{x^2}{2}}$$

Now, $F(1) = \frac{1}{2}$

* for $1 \leq x \leq 2$

$$F_X(x) = \frac{1}{2} + \int_1^x (2-t)\,dt$$

$$= \frac{1}{2} + \left[2t - \frac{t^2}{2}\right]_1^x$$

$$= \frac{1}{2} + \left[\left(2x - x^2/2\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$F_X(x) = \frac{1}{2} + 2x - x^2/2 - \frac{3}{2}$$

$$F_X(x) = \boxed{2x - \frac{x^2}{2} - 1}$$

Now, $F(2) = 2(2) - \frac{4}{2} - 1 = \boxed{1}$ as we expect

$$\therefore F_X(x) = \begin{cases} 0 & , x < 0 \\ \frac{x^2}{2} & , 0 \leq x \leq 1 \\ 2x - x^2/2 - 1 & , 1 \leq x \leq 2 \\ 1 & , x > 2 \end{cases}$$