Solution of the first examination	Math 106 Semester I	1442-1443

1. $S_4 = \frac{3-1}{12}\left(f(1) + 4f(\frac{3}{2}) + 2f(2) + 4f(\frac{5}{2}) + f(3)\right) \approx 4.505$ **(1.5)+(0.5)**

2.
$$\int \frac{(1-\frac{1}{x^2})^5}{x^3}dx \overset{t=1-\frac{1}{x^2}}{=} \frac{1}{2}\int t^5 dt \qquad \textbf{(2)}$$
$$= \frac{1}{12}(1-\frac{1}{x^2})^6 + c. \qquad \textbf{(1)}$$

3. $\ln(y) = \frac{1}{2}\ln x + \frac{1}{3}\ln(x+2) + \frac{1}{5}\ln(x-1).$ **(1.5)**
Then $\frac{y'}{y} = \frac{1}{2x} + \frac{1}{3(x+2)} + \frac{1}{5(x-1)}.$
So $y' = \left(\frac{1}{2x} + \frac{1}{3(x+2)} + \frac{1}{5(x-1)}\right)\sqrt{x}.\sqrt[3]{x+2}.\sqrt[5]{x-1}.$ **(0.5)**

4.
$$\int \frac{(\sec x)^2}{\sqrt{4-(\tan x)^2}}dx \overset{t=\tan x}{=} \int \frac{dt}{\sqrt{4-t^2}} \qquad \textbf{(1)}$$
$$= \sin^{-1}(\frac{t}{2}) + c = \sin^{-1}(\frac{\tan x}{2}) + c. \qquad \textbf{(1)}$$

5.
$$\int \frac{dx}{\sqrt{e^{2x}-1}} \overset{t=e^x}{=} \int \frac{dt}{t\sqrt{t^2-1}} \qquad \textbf{(2)}$$
$$= \sec^{-1}(e^x) + c. \qquad \textbf{(1)}$$

6.
$$\int \frac{dx}{x\sqrt{1-x^5}} \overset{t^2=x^5}{=} \frac{2}{5}\int \frac{dt}{t\sqrt{1-t^2}} \qquad \textbf{(2)}$$
$$= -\frac{2}{5}\operatorname{sech}^{-1}(x^{\frac{5}{2}}) + c \qquad \textbf{(1)}$$

7.
$$\lim_{x\to 0}\frac{\cos x - 1 + \frac{x^2}{2}}{x^4} = \lim_{x\to 0}\frac{-\sin x + x}{4x^3} \qquad \textbf{(1)}$$
$$= \lim_{x\to 0}\frac{-\cos x + 1}{12x^2} \qquad \textbf{(1)}$$
$$= \lim_{x\to 0}\frac{\sin x}{24x} = \frac{1}{24}. \qquad \textbf{(1)}$$

8.

$$\begin{aligned}\int (\ln x)^2 dx &\overset{u=(\ln x)^2, v'=1}{=} x(\ln x)^2 - 2\int \ln x dx \quad \mathbf{(1.5)}\\ &= x(\ln x)^2 - 2x\ln x + 2x + c. \quad \mathbf{(1.5)}\end{aligned}$$

9.

$$\begin{aligned}\int (\tan x)^5(\sec x)^3 dx &\overset{t=\sec x}{=} \int (t^2-1)^2 t^2 dt \quad \mathbf{(1)}\\ &= \frac{1}{7}\sec^7 x - \frac{2}{5}\sec^5 x + \frac{1}{3}\sec^3 x + c. \quad \mathbf{(2)}\end{aligned}$$

10.

$$\begin{aligned}\int \frac{x^2}{\sqrt{9-x^2}} dx &\overset{x=3\sin\theta}{=} 9\int \sin^2\theta d\theta = \frac{9}{2}\int (1-\cos(2\theta))d\theta \quad \mathbf{(1)}\\ &= \frac{9}{2}(\theta - \sin\theta\cos\theta) + c \quad \mathbf{(0.5)}\\ &= \frac{9}{2}\left(\sin^{-1}(\frac{x}{3}) - \frac{1}{9}x\sqrt{9-x^2}\right) + c \quad \mathbf{(1.5)}\end{aligned}$$

11.

$$\begin{array}{r|l} x^2+8x+10 & x^2+6x+11 \\ \hline -x^2-6x-11 & 1 \\ \hline 2x \quad -1 & \end{array}$$

$$\begin{aligned}\int \frac{x^2+8x+10}{x^2+6x+11} dx &= \int 1 + \frac{2x-1}{x^2+6x+11} dx \quad \mathbf{(1)}\\ &= \int 1 + \frac{2x+6}{x^2+6x+11} dx - \int \frac{7}{(x+3)^2+2} dx\\ &= x + \ln\left(x^2+6x+11\right) - \frac{7}{\sqrt{2}}\tan^{-1}\frac{(x+3)}{\sqrt{2}} + c. \quad \mathbf{(2)}\end{aligned}$$