# 106Midterm2grading scheme(Sem1-40/41)

**Question1** a) $y = (1+8x^2)^{\frac{1}{x^2}}$

$$\lim_{x\to 0} lny = \lim_{x\to 0}\frac{\ln(1+8x^2)}{x^2} = \lim_{x\to 0}\frac{16x}{(1+8x^2)(2x)} = 8 \quad (\mathbf{1,5})$$

So $\lim_{x\to 0} y = e^8 \quad (\mathbf{0,5})$

b) $\int e^{4x} sinxdx = \frac{1}{4}e^{4x}sinx - \frac{1}{4}\int e^{4x}cosxdx \quad (\mathbf{1})$

$$= \frac{1}{4}e^{4x}sin4x - \frac{1}{4}(\frac{1}{4}e^{4x}cosx + \frac{1}{4}\int e^{4x}sinxdx) \quad (\mathbf{1})$$

Thus $\int e^{4x}sin4xdx = \frac{1}{17}e^{4x}(4sinx - cosx) + C \quad (\mathbf{1})$

c) $\int (sinx)^2(cosx)^2dx = \frac{1}{4}\int (sin2x)^2dx = \frac{1}{8}\int (1-cos4x)dx \quad (\mathbf{2})$

$$= \frac{1}{8}\left(x - \frac{sin4x}{4}\right) + C \quad (\mathbf{1})$$

**Question2** a) $\int \frac{\sqrt{x^2-25}}{x}dx = 5\int (tan\theta)^2 d\theta \quad x = 5sec\theta \quad (\mathbf{1})$

$$= 5(tan\theta - \theta) + C \quad (\mathbf{1})$$

$$= \sqrt{x^2-25} - 5\sec^{-1}\left(\frac{x}{5}\right) + C \quad (\mathbf{1})$$

$$b)\ \frac{3x^2+7x+2}{(x+1)^2(x+3)} = \frac{1}{x+1} - \frac{1}{(x+1)^2} + \frac{2}{x+3} \quad (\mathbf{1.5})$$

$$\int \frac{3x^2+7x+2}{(x+1)^2(x+3)}dx = \ln|x+1| + \frac{1}{x+1} + 2\ln|x+3| + C \quad (\mathbf{1.5})$$

c) $\int \frac{dx}{\sqrt{x^2 + x + 1}} = \int \frac{dx}{\sqrt{(x + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}}$ $(\mathbf{1})$

$= \sinh^{-1}\left(\frac{(2x+1)}{\sqrt{3}}\right) + C$ $(\mathbf{1})$

## Question3

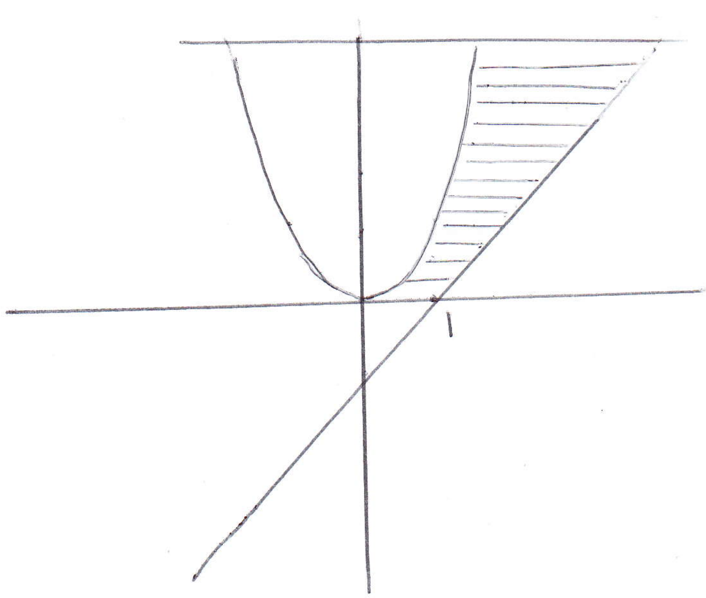
a) $\int \frac{dx}{x^{1/2}+x^{1/3}} = 6\int \frac{u^3 du}{1+u} = 6\int (u^2 - u + 1 - \frac{1}{u+1})du$ $(\mathbf{2})$

$= 2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln\left|1 + x^{\frac{1}{6}}\right| + C$ $(\mathbf{1})$

b$\int_0^c \frac{xdx}{1+x^4} = \frac{1}{2}\int_0^{c^2} \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}(c^2) \to \frac{\pi}{4}$ $as\ c \to \infty$ $(\mathbf{2.5})$

Thus $\int_0^\infty \frac{xdx}{1+x^4}$ $converges\ and\ is\ equal\ to\ \frac{\pi}{4}$ $(\mathbf{0.5})$

c) graph$(\mathbf{1})$



$$A = \int_0^4 (y + 1 - \sqrt{y})dy = [\frac{y^2}{2} + y - \frac{2}{3}y^{\frac{3}{2}}]_0^4 = \frac{20}{3} \quad \textbf{(2)}$$