**King Saud University: Mathematics Department Math-254**
**First Semester 1445 H Final Examination**
**Maximum Marks = 40 Time: 180 mins.**

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**Name of the Student:**—————————— **I.D. No.** ——————————

**Name of the Teacher:**—————————— **Section No.** ——————————

# Note: Check the total number of pages are Six (6).

(**15 Multiple choice questions and Two (2) Full questions**)

**The Answer Tables for Q.1 to Q.15 :** Marks: 2 for each one ($2 \times 15 = 30$)

Ps. : Mark {a, b, c or d} for the correct answer in the box.

| Q. No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| a,b,c,d | | | | | | | | | | |

| Q. No. | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|
| a,b,c,d | | | | | |

| Quest. No. | Marks Obtained | Marks for Questions |
|---|---|---|
| Q. 1 to Q. 15 | | 30 |
| Q. 16 | | 5 |
| Q. 17 | | 5 |
| Total | | 40 |

**Question 1:** If $x_{n+1} = \dfrac{a}{b - \cos(x_n)}$, $n \geq 0$, is the fixed-point iterative form of the nonlinear equation $\dfrac{2}{x} + \cos(x) - 3 = 0$, then the value of the constants $a$ and $b$ are:

**(a)** $a = 2, b = 1$ (b) $a = 3, b = 2$ (c) $a = 2, b = 3$ (d) None of these

**Question 2:** The next iterative value of the root of $x^3 = 3x - 2$ using the secant method, if the initial guesses are $-2.6$ and $-2.4$ is:

**(a)** $-2.2066$ (b) $-2.1066$ (c) $-2.3066$ (d) None of these

**Question 3:** If the iterative scheme $x_{n+1} = x_n - k\dfrac{f(x_n)}{f'(x_n)}$, $n \geq 0$, converges at least quadratic to a simple root $\alpha$, than the value of $k$ is:

**(a)** k=1 (b) k=2 (c) k=3 (d) None of these

**Question 4**: The $l_\infty-$norm of the inverse of the Jacobian matrix for the nonlinear system $x^2 + y^2 = 4,\ 2x - y^2 = 0$ using $[x_0, y_0]^t = [1, 1]^t$ is:

**(a)** 2 (b) 0.5 (c) 4 (d) None of these

**Question 5**: Let $A = \begin{bmatrix} 1.001 & 1.5 \\ 2 & 3 \end{bmatrix}$, then the determinant of a lower-triangular matrix $L$ of the LU factorization using Crouts method is:

(a) 0.300 (b) 1.001 (c) 0.003 (d) None of these

**Question 6**: The $l_\infty-$norm of the Jacobi iteration matrix of the following linear system $4x_1 - x_2 + x_3 = 7,\ 4x_1 - 8x_2 + x_3 = -21,\ -2x_1 + x_2 + 5x_3 = 15$ is:

(a) 0.625 (b) 0.5 (c) 0.4 (d) None of these

**Question 7**: Using Gauss-Seidel method and starting with $\mathbf{x}^{(0)} = [1.200, 0.467, 1.033]^t$, then the first approximation of the solution for the following linear system is:
$5x_1 + 2x_2 - x_3 = 6,\ x_1 + 6x_2 - 3x_3 = 4,\ 2x_1 + x_2 + 4x_3 = 7$ is:

(a) $\mathbf{x}^{(1)} = \begin{pmatrix} 1.024 \\ 1.006 \\ 0.987 \end{pmatrix}$ (b) $\mathbf{x}^{(1)} = \begin{pmatrix} 0.897 \\ 0.950 \\ 1.019 \end{pmatrix}$ (c) $\mathbf{x}^{(1)} = \begin{pmatrix} 1.220 \\ 0.980 \\ 0.895 \end{pmatrix}$ (d) None of these

**Question 8**: Let $A = \begin{bmatrix} 0 & \alpha \\ 1 & 1 \end{bmatrix}$ and $1 < \alpha < 2$. If the condition number $k(A)$ of the matrix $A$ is 6, then $\alpha$ equals to

(a) 0.8 (b) 0.5 (c) 0.2 (d) None of these

**Question 9:** Let $x_0 = 2$, $x_1 = 2.5$, $x_2 = 4$ and $x_3 = 5.5$. If the best approximation of $f(x) = \dfrac{1}{x}$ at $x = 3$ using quadratic interpolation formula is $P_2(3) = 0.325$, then the value of the unknown point $\eta$ in the error formula is equal to :

**(a)** 2.9201 **(b)** 2.7859 **(c)** 3.1472 **(d)** None of these

**Question 10:** If $x_0 = 0$, $x_1 = 1$, $x_2 = 3$ and for a function $f(x)$, the divided differences are $f[x_1] = 2$, $f[x_2] = 3$, $f[x_0, x_1] = 1$, $f[x_1, x_2] = \frac{1}{2}$, $f[x_0, x_1, x_2] = -\frac{1}{6}$. Then the approximation of $f(\frac{1}{2})$ using quadratic interpolation Newton formula is:

**(a)** 1.5417 **(b)** 4.1232 **(c)** 2.3481 **(d)** None of these

**Question 11:** Let $f(x) = x^3$ and $h = 0.1$. The absolute error for the approximation of $f'(0.2)$ using 2-point forward difference formula is:

**(a)** 0.0722 **(b)** 0.0711 **(c)** 0.0700 **(d)** None of these

**Question 12:** The absolute error for the approximation of the integral $\displaystyle\int_1^2 \frac{1}{x+1}\, dx$ using simple Trapezoidal's rule is:

(a) 0.1120 (b) 0.0112 (c) 0.0012 (d) None of these

**Question 13:** The approximation to the integral $\displaystyle\int_0^2 e^x\, dx$ using simple Simpson's rule is:

(a) 8.4207 (b) 7.4207 (c) 6.4207 (d) None of these

**Question 14:** For the initial value problem, $(x+1)y' + y^2 = 0, y(0) = 1, n = 1$, if the actual solution of the differential equation is $y(x) = \dfrac{1}{(1 + \ln(x+1))}$, then the absolute error by using Euler's method for the approximation of $y(0.05)$ is:

**(a)** 0.0350 **(b)** 0.0035 **(c)** 0.0042 **(d)** None of these

**Question 15:** Using the Taylor's method of order 2 to find the approximate value of $y(0.1)$ for the initial-value problem, $y' = e^{-2x} - 2y,\ y(0) = 0.1, n = 1$, is:

**(a)** 0.1620 **(b)** 0.1983 **(c)** 0.1846 **(d)** None of these

**Question 16:** Let $f(x) = \dfrac{3^x}{x}$ and $h = 0.1$ Compute the approximate value of $f''(3)$ and the absolute error. If $\max|f^{(4)}| = 6.1022$, then find the number of subintervals required to obtain the approximate value of $f''(3)$ within the accuracy $10^{-4}$.

**Question 17:** Determine the number of subintervals required to approximate the integral $\int_0^2 \frac{1}{x+4}dx$, with an error less than $10^{-4}$ using composite Simpson's rule. Then approximate the given integral and compute the absolute error.