

Question 2 :

a)

$$\int \frac{dx}{x\sqrt{x^4-1}} \stackrel{t=x^2}{=} \frac{1}{2} \int \frac{dt}{t\sqrt{t^2-1}} \quad (2)$$

$$= \frac{1}{2} \sec^{-1}(x^2) + c. \quad (1)$$

b)

$$\int \frac{\cot x dx}{\sqrt{1-\sin^4 x}} \stackrel{t=\sin^2 x}{=} \frac{1}{2} \int \frac{dt}{t\sqrt{1-t^2}} \quad (2)$$

$$= -\frac{1}{2} \operatorname{sech}^{-1}(\sin^2 x) + c. \quad (1)$$

c)

$$\int \cosh^{-1} x dx = x \cosh^{-1} x - \int \frac{x}{\sqrt{x^2-1}} dx \quad (2)$$

$$= x \cosh^{-1} x - \sqrt{x^2-1} + c. \quad (1)$$

Question 3 :

a)

$$\int \frac{dx}{(x^2+4)^2} \stackrel{x=2\tan\theta}{=} \frac{1}{8} \int \cos^2 \theta d\theta$$

$$= \frac{1}{16} (\theta + \sin \theta \cos \theta) + c \quad (2)$$

$$= \frac{1}{16} \left(\tan^{-1}\left(\frac{x}{2}\right) + \frac{2x}{x^2+4} \right) + c. \quad (1)$$

b)

$$\int \frac{dx}{x^{\frac{1}{2}} - x^{\frac{1}{3}}} \quad x=t^6 = 6 \int \frac{t^3}{t-1} dt = 6 \int \left(t^2 + t + 1 + \frac{1}{t-1} \right) ds \quad (2)$$

$$= 2\sqrt{x} + 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} + 6 \ln |x^{\frac{1}{6}} - 1| + c. \quad (1)$$

c)

$$\int \frac{dx}{5 + 3 \cos x + 4 \sin x} \quad t = \tan\left(\frac{x}{2}\right) = \int \frac{dt}{(t+2)^2} \quad (2)$$

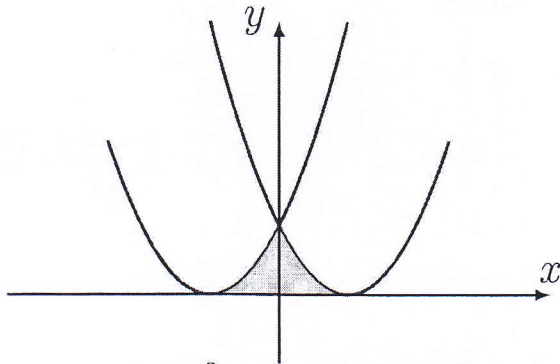
$$= -\frac{1}{\tan\left(\frac{x}{2}\right) + 2} + c. \quad (1)$$

Question 4 :

$$a) \int_0^a (1+x)e^{-2x} dx = \frac{3}{4} - \frac{3}{4}e^{-2a} - \frac{a}{2}e^{-2a} \quad (2).$$

$$\text{So the integral converges and } \int_0^{+\infty} (1+x)e^{-2x} dx = \frac{3}{4}. \quad (1)$$

b) Graph: (1)



$$A = 2 \int_{-1}^0 (x-1)^2 dx = \frac{2}{3}. \quad (2)$$

Question 5 :

The volume obtained by revolving the region bounded by $y = x^2$ and $y = \sqrt{x}$ about the x -axis is

$$V = \pi \int_0^1 (x - x^4) dx = \pi \left(\frac{1}{2} - \frac{1}{5} \right) = \frac{3\pi}{10}.$$

(Intersection: (0.5))

Integral: (2)

result (0.5)).

Question 6 :

a)

$$SA = 2\pi \int_0^1 4t\sqrt{16t^2 + 16} dt = 32\pi \int_0^1 t\sqrt{t^2 + 1} dt. \quad (1.5)$$

$$= 16\pi \int_1^2 \sqrt{u} du \quad (1)$$

$$= \frac{32\pi}{3} \pi (2\sqrt{2} - 1) \quad (0.5)$$

b) Graph: (1)

