Solution of the first examination Math 106Semester I 1441-1442

1. $f(x) = \sin^4(x)$, $n = 6$

$$S_6 = \frac{2\pi - 0}{3.6}\left(0 + 4\sin^4\frac{\pi}{3} + 2\sin^4\frac{2\pi}{3} + 0 + 2\sin^4\frac{4\pi}{3} + 4\sin^4\frac{5\pi}{3} + 0\right).$$

$$\int_0^{\pi} \sin^4(x)dx \approx \frac{3\pi}{4}.$$

2.

$$\begin{aligned}\int \frac{\ln(x)+1}{1+(x\ln x)^2}dx &\overset{u=x\ln x}{=} \int \frac{du}{1+u^2} \qquad (2)\\ &= \tan^{-1}(x\ln x) + c. \qquad (1)\end{aligned}$$

3.

$$\begin{aligned}\int \frac{x+\sin^{-1}x}{\sqrt{1-x^2}}dx &= \int \frac{x}{\sqrt{1-x^2}}dx + \int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx\\ &= -\sqrt{1-x^2} + \frac{1}{2}(\sin^{-1}x)^2 + c\\ &\quad (1.5) \qquad\qquad (1.5)\end{aligned}$$

4.

$$\begin{aligned}\int \frac{x^2}{\sqrt{1+x^6}}dx &= \frac{1}{3}\int \frac{3x^2}{\sqrt{1+x^6}}dx \qquad (0.5)\\ &= \frac{1}{3}\sinh^{-1}(x^3) + c \qquad (1.5)\end{aligned}$$

5.

$$\begin{aligned}\int \frac{dx}{\sqrt{4-e^x}} &\overset{u^2=e^x}{=} 2\int \frac{du}{u\sqrt{4-u^2}} \qquad (2)\\ &= -\text{sech}^{-1}(\frac{e^{\frac{x}{2}}}{2}) \qquad (1)\end{aligned}$$

6.

$$\begin{aligned}\int \sin^5 x\cos^2 x dx &\overset{u=\cos x}{=} -\int (1-u^2)^2 u^2 du \qquad (2)\\ &= -\frac{1}{3}\cos^3 x - \frac{1}{7}\cos^7 x + \frac{2}{5}\cos^5 x + c \qquad (1)\end{aligned}$$

7. $Y = (1+\frac{7}{x})^{5x}$, $\ln Y = \frac{\ln(1+\frac{7}{x})}{\frac{1}{5x}} \underset{x\to+\infty}{\longrightarrow} 35.$ (2.5)

$\lim_{x\to+\infty}(1+\frac{7}{x})^{5x} = e^{35}.$ (0.5)

8.

$$\begin{aligned}\int x^2 \log_3 x dx &= \frac{1}{\ln 3}\int x^2 \ln x dx = \frac{1}{\ln 3}\left(\frac{x^3}{3}\ln x - \frac{1}{3}\int x^2 dx\right) \qquad (2)\\ &= \frac{1}{3\ln 3}\left(x^3 \ln x - \frac{x^3}{3}\right) + c \qquad (1)\end{aligned}$$

9.

$$\begin{aligned}\int \frac{x^2}{\sqrt{3-x^2}} dx &\overset{x=\sqrt{3}\sin\theta}{=} 3\int \sin^2\theta d\theta\\ &= \frac{3}{2}(\theta - \sin\theta\cos\theta) + c \qquad (2)\\ &= \frac{3}{2}(\sin^{-1}(\frac{x}{\sqrt{3}}) - \frac{x\sqrt{3-x^2}}{3}) + c \qquad (1)\end{aligned}$$

10. $y = (\cos x)^x$, $\frac{y'}{y} = \ln\cos x - \frac{x\sin x}{\cos x}$ (1.5)

$y' = \left(\ln\cos x - \frac{x\sin x}{\cos x}\right) y$ (0.5)

11. $\dfrac{7x+10}{(x+1)(x^2-4)} = -\dfrac{1}{x+1} + \dfrac{2}{x-2} - \dfrac{1}{x+2}$ (2)

$\int\limits_{(1)} \dfrac{7x+10}{(x+1)(x^2-4)}dx = -\ln|x+1| + 2\ln|x-2| - \ln|x+2| + c$