106Midterm (Sem2/42)

Question1

a) P={0,1,2,3,4}

$$S_4 = \frac{4-0}{3.4}\left\{\sqrt{8} + 12 + 8 + 4\sqrt{35} + \sqrt{72}\right\}$$

$$so \int_0^4 \sqrt{8+x^3}\ dx \approx 18.326$$

b) $\int \frac{e^x \ln(e^x+1)dx}{e^x+1} = \int u du \quad u = \ln(1+e^x)$

$$= \frac{1}{2}(ln(1+e^x))^2 + C$$

c) $y = (1+x^2)^{2x+1}$

$$\frac{y'}{y} = 2\ln(1+x^2) + \frac{2x(2x+1)}{1+x^2}$$

$$y' = \left(2\ln(1+x^2) + \frac{2x(2x+1)}{1+x^2}\right)y$$

Question2

a) $\int \frac{2^x dx}{\sqrt{4-4^x}} = \frac{1}{ln\,2}\int \frac{du}{\sqrt{2^2-u^2}} \quad u = 2^x$

$$= \frac{1}{ln\,2}\sin^{-1}\left(\frac{2^x}{2}\right) + C$$

b) $\int \frac{x^4 dx}{\sqrt{x^{10}-1}} = \frac{1}{5}\int \frac{du}{\sqrt{u^2-1}} \quad u = x^5$

$$= \frac{1}{5}\cosh^{-1}(x^5) + C$$

c) $\int \frac{dx}{x\sqrt{1-x^6}} = \frac{1}{3}\int \frac{du}{u\sqrt{1-u^2}} \quad u = x^3$

$$= \frac{-1}{3}\operatorname{sech}^{-1}(x^3) + C$$

Question3

a) $\lim_{x\to\infty} \frac{\ln(1+3x)}{x} = \lim_{x\to\infty} \frac{3}{1+3x} = 0$

$so \lim_{x\to\infty} (1+3x)^{\frac{1}{x}} = 1$

b) $\int e^x \, sin4xdx = e^x sin4x - 4\int e^x cos4xdx$

$$\int e^x \, cos4xdx = e^x cos4x + 4\int e^x sin4xdx$$

$$so \int e^x \, sin4xdx = \frac{1}{17} e^x (sin4x - 4cos4x) + C$$

c) $\int (cosx)^5 (sinx)^4 dx = \int (1-u^2)^2 u^4 du \quad u = sinx$

$$= \frac{(sinx)^9}{9} - \frac{2}{7}(sinx)^7 + \frac{1}{5}(sinx)^5 + C$$

Question4

a) $\int x^3 \sqrt{x^2-4}dx = 32\int (sec\theta)^4 (tan\theta)^2 d\theta \quad x = 2sec\theta$

$$= 32\int (1+u^2)u^2 du \quad u = tan\theta$$

$$= \frac{32}{5}(tan\theta)^5 + \frac{32}{3}(tan\theta)^3 + C$$

$$= \frac{1}{5}(\sqrt{x^2-4})^5 + \frac{4}{3}(\sqrt{x^2-4})^3 + C$$

b) $\frac{2x^2-11x+9}{x^3-6x^2+9x} = \frac{1}{x} + \frac{1}{x-3} - \frac{2}{(x-3)^2}$

$$\int \frac{2x^2-11x+9}{x^3-6x^2+9x} dx = \ln|x| + \ln|x-3| + \frac{2}{x-3} + C$$