King Saud University: Mathematics Department Math-254
Third Semester 1444 H Midterm Exam.
Maximum Marks = 30 Time: 120 mins.

Question 1: Show that the one of the possible rearrangement of the nonlinear equation $e^x - x^2 = 2 - 3x$, which has root in $[0, 0.6]$ is

$$x_{n+1} = g(x_n) = \frac{2 - e^x_n + x_n^2}{3}; \qquad n = 0, 1, \ldots.$$

Show that $g(x)$ has a unique fixed-point in $[0, 0.6]$. Determine the number of iterations needed to get an approximation with accuracy 10^{-5} to the solution of $g(x) = \dfrac{2 - e^x + x^2}{3}$ lying in the interval $[0, 0.6]$ by using the fixed-point iteration method and $x_0 = 0.5$. [6 Marks]

Question 2: Show that the rate of convergence of Newton's method at the root $\alpha = 1$ of the equation $(x-1)^2 \sin x = 0$ is linear. Use quadratic convergence method to find x_2 using $x_0 = 1.5$. Compute the relative error. [6 Marks]

Question 3: Find the values of a, b and c such that the iterative scheme

$$x_{n+1} = ax_n + \frac{bN}{x_n^2} + \frac{cN^2}{x_n^5}, \qquad n \geq 0,$$

converges at least cubically to $\alpha = N^{\frac{1}{3}}$. Use this scheme to find second approximation of $(8)^{\frac{1}{3}}$ when $x_0 = 1.8$. [6 Marks]

Question 4: Use the simple Gaussian elimination method to find the inverse A^{-1} of the following matrix

$$A = \begin{pmatrix} 1 & -1 & 2\alpha \\ 2 & 2 & 4 \\ 0 & 4 & 8 \end{pmatrix}.$$

by using largest negative integer value of α. Use A^{-1} to find the unique solution of the system $A\mathbf{x} = [-4, 8, 12]^T$. [6 Marks]

Question 5: Use LU decomposition by Dollittle's method to find the value(s) of nonzero α for which the linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} \alpha & 4 & 1 \\ 2\alpha & -1 & 2 \\ 1 & 3 & \alpha \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 6 \\ 3 \\ 5 \end{pmatrix},$$

is inconsistent and consistent. Solve the consistent system. [6 Marks]