MID TERM EXAMINATION, SEMESTER I, 1443
DEPARTMENT OF MATHEMATICS, COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH - 107 FULL MARK: 30 TIME: 2 HOURS

[N. B.: Marks: Q1. [5]; Q2. [5]; Q3.[2+2+2=6]; Q4. [2+3+3=8] Q5. [2+4=6]]

Q1. Solve the system of linear equations by Gaussian elimination:

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 5 \\ x_1 + x_2 + x_3 &= 2 \\ 4x_1 + 3x_2 - 3x_3 &= 11 \end{aligned}$$

Q2. Let

$$A = \begin{bmatrix} 1 & -1 & \alpha \\ -1 & 2 & -\alpha \\ \alpha & 1 & 1 \end{bmatrix}$$

Find the values of α for which the matrix A is invertible.

Q3. (i) Find the angle between the vectors $\mathbf{u} = 10\mathbf{i} + 9\mathbf{j}$ and $\mathbf{v} = -4\mathbf{i} + 2\mathbf{j}$.
(ii) Let $\mathbf{a} = \langle 1, 0, 0\rangle$ and $\mathbf{b} = \langle -6, 2, 1\rangle$. Determine the component of $\mathbf{b}$ along $\mathbf{a}$.
(iii) Determine whether $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j}$ are perpendicular to each other.

Q4. (a) Find the work done that is exerted by a constant force $\mathbf{F} = 3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ to move a particle from a point $P(-1, 1, 2)$ to another point $Q(2, 4, 3)$.
(b) Find the equation of the plane determined by the points $P(4, -3, 1)$, $Q(6, -4, 7)$ and $R(1, 2, 2)$.
(c) Sketch the graph of the equation $4x^2 - 9y^2 + z^2 = 36$ in an xyz-coordinate system, and identify the surface.

Q5. Let C be the curve determined by $\mathbf{r}(t) = (1 + t^3)\mathbf{i} + \sqrt{2t - 1}\,\mathbf{j} + t^3\mathbf{k}$.
(a) Find the domain of $\mathbf{r}(t)$.
(b) Find parametric equations for the tangent line to C at the point $(2, 1, 1)$.