Name: **ID:** **Section:** 54030 **Mark:**

King Saud University

College of Sciences, Department of Mathematics

1444/Semester-2/ MATH 380/ Quiz-1

Marks: 10 **Max. Time: 35 Minutes**

Answer the following questions.

Q1: [2.5+2.5]

(a) A fraction $p = 0.05$ of the items coming off a production process are defective. If a random sample of 10 items is taken from the output of the process, what is the probability that the sample contains exactly one defective item? What is the probability that the sample contains one or fewer defective items?

(b) Let X and Y two random variables have the joint normal distribution. What value of α that minimizes the variance of $Z=\alpha X+(1-\alpha)Y$? Simplify your result when X and Y are independent.

Q2: [2.5+2.5]

(a) The lifetime T of a certain component has an exponential distribution with parameter $\lambda=0.02$. Find $\Pr(T \le 120 | T > 100)$

(b) Suppose that X is a Poisson distributed random variable with mean $\lambda = 2$. Determine $\Pr\{X \le \lambda\}$.

The Model Answer

Q1: [2.5+2.5]

(a) If $X \sim Bin(n,p)$ then $\Pr\{X=x\} = \binom{n}{x} p^x q^{n-x},\ 0<p<1,\ q=1-p$

(i)

$$\Pr\{X=1\} = \binom{10}{1} 0.05^1 0.95^9$$

$$\simeq 0.31512$$

(ii)

$$\because \Pr\{X \le 1\} = \Pr\{X=0\} + \Pr\{X=1\}$$

$$= \binom{10}{0} 0.05^0 0.95^{10} + \binom{10}{1} 0.05^1 0.95^9$$

$$\therefore \Pr\{X \le 1\} \simeq 0.9139$$

(b)

$$\mathrm{Z} = \alpha \mathrm{X} + (1-\alpha)\mathrm{Y}$$

$$\mathrm{Var(Z)} = \alpha^2 \sigma_X^2 + 2\alpha(1-\alpha)\rho\sigma_X\sigma_Y + (1-\alpha)^2\sigma_Y^2$$

To get α^* that minimizes Var(Z) let $\frac{\partial V}{\partial \alpha} = 0$

$$\therefore\ 2\alpha\sigma_X^2 + (2-4\alpha)\rho\sigma_X\sigma_Y + (-2+2\alpha)\sigma_Y^2 = 0$$

$\Rightarrow$

$$\alpha^* = \frac{\sigma_Y^2 - \rho\sigma_X\sigma_Y}{\sigma_X^2 - 2\rho\sigma_X\sigma_Y + \sigma_Y^2}$$

If X and Y are independent random variables, then $\rho = 0$

Consequently $\alpha^* = \frac{\sigma_Y^2}{\sigma_X^2 + \sigma_Y^2}$.

Q2: [2.5+2.5]

(a)

$$\Pr(T \le 120 | T > 100) = 1 - pr(T > 120 | T > 100)$$

$$= 1 - pr(T > 20)$$

$$= 1 - \mathrm{e}^{-0.02(20)}$$

$$\therefore \Pr(T \le 120 | T > 100) = 1 - \mathrm{e}^{-0.4} \simeq 0.33$$

(b)

$$\because \Pr\left\{X \le 2\right\} = \sum_{x=0}^{2} \frac{e^{-2} 2^{x}}{x!}$$

$$= e^{-2}\left[1 + \frac{2}{1!} + \frac{2^2}{2!}\right]$$

$$\therefore \Pr\left\{X \le 2\right\} = 5e^{-2} \simeq 0.6767$$