



Choose 5 questions of the following:

Q1: [2.5+2.5]

A company has 3 suppliers, designated A, B, and C. The relative amounts of a certain product purchased from each of the suppliers are 50%, 35% and 15%, respectively. If the proportion defective produced by each supplier are 1%, 2% and 3%, respectively.

- Find the overall proportion defective of that product.
- If a defective random product is selected, find the probability that this product was made by supplier A. **Hint:** use Bay's theorem.

Q2: [2.5+2.5]

- For $X \sim \text{Poisson}(\lambda)$, use the moment generating function to derive the first moment of X .

Hint: use the expansion $e^y = \sum_{x=0}^{\infty} \frac{y^x}{x!}$

- A car insurance company receives 3 claims on the average per day. Assuming that the number of claims follows a Poisson distribution, what is the probability that the company receives at least 4 claims on a randomly selected day?

Q3: [5]

Suppose that the price X of a particular stock at closing has a log-normal distribution with mean \$30 and a variance 5. What is the probability that the price exceeds \$35?

Q4: [2 + 2 +1]

Let X and Y be random variables with joint pdf given by

$$f_{X,Y}(x, y) = \begin{cases} 6(1 - y), & 0 \leq x \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Find $P\left(X \leq \frac{3}{4}, Y \geq \frac{1}{2}\right)$.
- (b) Find $f_X(x)$ and $f_Y(y)$.
- (c) Are X and Y independent?

Q5: [5]

In some health insurance company, loss amounts are uniformly distributed on the interval [0,450]. Assume a deductible d applies for every loss, and the expected value of payment made by the insurance company equals 120. Calculate d.

Q6: [2+2+1]

Let X and Y be two random variables with joint density function

$$f_{X,Y}(x, y) = \begin{cases} cxy & , \quad 0 \leq y \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Determine the value of c.
- (b) Find $f_X(x)$ and $f_{Y|X}(y|x)$.
- (c) Find $E[Y|X = x]$.

Table 5.1 Area $\Phi(x)$ Under the Standard Normal Curve to the Left of X .

X	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998