King Saud University: Mathematics Department Math-254
First Semester 1445 H Second Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Question 1: Use LU-factorization method with Doolittle's method ($l_{ii} = 1$) to find the solution of the consistent system for $\alpha \neq 3$. [7 Marks]

$$\begin{aligned} x_1 + x_2 &= 1 \\ 3x_1 + \alpha x_2 + 5x_3 &= 8 \\ 7x_2 + 3x_3 &= 3 \end{aligned}$$

Question 2: Consider the following linear system of equations [6 Marks]

$$\begin{aligned} 2x_1 + x_2 &= 3 \\ x_1 + 8x_2 + x_3 &= 10 \\ x_2 + 2x_3 &= 3 \end{aligned}$$

If $\mathbf{x} = [1, 1, 1]^T$ be the exact solution of the system, then using Jacobi iterative method and $\mathbf{x}^{(0)} = [0.5, 0.5, 0.5]^T$, compute the absolute error $\|\mathbf{x} - \mathbf{x}^{(2)}\|$. How many iterations needed to get an accuracy within 10^{-4} using Jacobi iterative method.

Question 3: Consider a linear system $A\mathbf{x} = \mathbf{b}$, where [6 Marks]

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 4 & 0 \\ 1 & 2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

If $\mathbf{b}$ is changed to $\mathbf{b}^* = [1, 1, 1.99]^T$, then use the residual vector $\mathbf{r}$ to find the relative error in the solution to the linear system $A\mathbf{x} = \mathbf{b}$.

Question 4: Use the following table to find the best approximation of $f(0.6)$ by using quadratic Lagrange interpolating polynomial for equally spaced data points [6 Marks]

x	0.15	0.2	0.3	0.5	0.55	0.8	1
$f(x)$	-0.0427	-0.0644	-0.1084	-0.1733	-0.1808	-0.1428	0

The above table is for $f(x) = x^2 \ln x$. Determine the number of points when the error for quadratic Lagrange interpolation for equally spaced data points is to be bounded by 10^{-6}.