King Saud University: Mathematics Department Math-254
First Semester 1445 H Makeup Second Midterm Exam.
Maximum Marks = 25 Time: 90 mins.

Questions: (7 + 6 + 6 + 6) Marks

Question 1: Construct the LU decomposition of the following matrix A by using the LU decomposition by Doolittle's method. Find the value(s) of α for which the following matrix

$$A = \begin{pmatrix} 1 & -1 & \alpha \\ -1 & 2 & -\alpha \\ \alpha & 1 & 1 \end{pmatrix},$$

is singular. Also, find the unique solution of the linear system $A\mathbf{x} = [1, 1, -1]^T$ by using the smallest positive integer value of α.

Question 2: Using LU decomposition by Crout's method, if the determinant of the following matrix A is -3, then find the unique solution of the linear system $A\mathbf{x} = [1, 1, 1]^T$, where

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & \alpha \end{pmatrix}.$$

Question 3: Consider the following nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 5 & 0 & -1 \\ -1 & 3 & 0 \\ 0 & -1 & 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

Find the matrix form of Gauss-Seidel iterative method. How many iterations needed to get an accuracy within 10^{-4}, using Gauss-Seidel iterative method and $\mathbf{x}^{(0)} = [0.5, 0.5, 0.5]^T$.

Question 4: Use the quadratic Lagrange interpolating polynomial by selecting the best three points from $\{-2, 0, 1, 2, 2.5\}$ on the function defined by $f(x) = (x + 1)^{\frac{1}{3}}$ to estimate the cube root of $\dfrac{3}{2}$ (that is, $(\dfrac{3}{2})^{\frac{1}{3}}$) and compute an error bound and the absolute error.