Department of Mathematics
College of Sciences
King Saud University, Riyadh.

M-203 (Differential and Integral Calculus)
1^{st} MidTerm Examination (1^{st} semester 1445) (2023/2024),
Time: 90 Minutes Max. Marks: 25.
Note: **All questions carry equal marks.**

Q1. Determine whether the sequence $\left\{\left(\frac{n+1}{n}\right)^{5n}\right\}$ converges or diverges, and if it converges find its limit.

Q2. Find the sum of the series:

$$\sum_{n=1}^{\infty}\left[\frac{5^n}{3^{2n}}+\frac{1}{(n+2)(n+3)}\right].$$

Q3. Determine whether the series $\sum_{n=1}^{\infty}(-1)^n\frac{3n}{2n^2+1}$ is absolutely convergent, conditionally convergent, or divergent.

Q4. Find the interval of convergence and the radius of convergence of the power series: $\sum_{n=1}^{\infty}\frac{(2x+6)^n}{n(-3)^n}$.

Q5. Find the Maclaurin series for the function $f(x)=\cos(x)$ and use it to approximate the integral $\int_0^1\frac{1-\cos(\sqrt{x})}{x}dx$ up to four decimal places.

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[illegible] (Differential and Integral Calculus)
[illegible] Examination [illegible]
Time: [illegible] Minutes Max. Marks: [illegible]
Note: All questions carry equal marks.

Q 1. Evaluate the double integral

$$\int_{0}^{[illegible]} \int_{[illegible]}^{[illegible]} y\sqrt{1+x^{3}}\,dx\,dy$$

Q 2. Use polar coordinates to evaluate the double integral

$$\int_{0}^{1} \int_{0}^{\sqrt{1-x^2}} \sin(x^2+y^2)\,dy\,dx.$$

Q 3. Find the surface area of the portion of the paraboloid $z = 9 - x^2 - y^2$ that lies above the xy-plane.

Q 4. Evaluate the integral:

$$\int_{0}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{1}^{2} 3(x+y)z\,dz\,dx\,dy.$$

Q 5. Evaluate the integral by changing to cylindrical coordinates

$$\int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{0}^{\sqrt{4-x^2-y^2}} z\,dz\,dy\,dx.$$

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M203 Final Examination/ First Sem. 1445

Max. Marks: 40 Marks: [Q1) 4+4+4, Q2) 4+4+4, Q3) 4+4+4+4] Time: 3 hs.

Q 1. (a) Determine whether the series $\sum_{n=2}^{\infty}(-1)^n\frac{1}{n(\ln n)^2}$ is absolutely convergent, conditionally convergent or divergent.

(b) Find the interval of convergence and the radius of convergence for the power series: $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{3^n n^2}(x-1)^n$.

(c) Find the power series representation of the function $f(x) = \tan^{-1}(x)$ and use its first three non-zero terms to approximate the value of $\tan^{-1}(0.1)$.

Q 2. (a) Evaluate the double integral: $\int_0^1\int_x^1 \sin(y^2)dydx$.

(b) Find the the first moment M_y about the y-axis of a lamina in the first quadrant having area mass density $\delta(x, y) = x^2 + y^2$ and which is bounded by the graphs of the equations: $y = 0, y = x$ and $x^2 + y^2 = 4$.

(c) Find the volume of the solid region inside the sphere $x^2+y^2+z^2 = 1$ and outside the cone $z = \sqrt{3x^2 + 3y^2}$.

Q 3. (a) Prove that the line integral $\int_C xdy + ydx$ is independent of path and evaluate the integral $\int_{(-1,1)}^{(1,1)} xdy + ydx$.

(b) Use Green's theorem to evaluate the integral $\oint_C y^3dx+(x^3+3xy^2)dy$, where C is the path from $(0, 0)$ to $(1, 1)$ along the graph $y = x^3$ and from $(1, 1)$ to $(0, 0)$ along the graph $y = x$.

(c) Use Divergence theorem to evaluate the integral $\iint_S \vec{F}\cdot\vec{n}\,dS$ for $\vec{F}(x, y, z) = (x, -y, z)$ and S is the surface $x^2 + y^2 + z^2 = 1$.

(d) Verify stoke's theorem for $\vec{F}(x, y, z) = (x, -y, z)$ and S is the surface of the hemisphere $z = \sqrt{9 - x^2 - y^2}$.