

<b>Question number</b>	I	II	III	IV	Total
<b>Mark</b>					

Question 1:

- (a) Let  $\sum_n x_n$  be a convergent series of real numbers. Prove that  $\lim_{n \rightarrow \infty} x_n = 0$ .
- (b) Is the converse of (a) true? Justify your answer.

(c) Test the following series for convergence:

(i)  $\sum_n (-1)^n \frac{\log n}{n}$  .

(ii)  $\sum \frac{1}{(n^3-1)}$

Question 2:

- (a) Prove that if a series is absolutely convergent, then it is convergent.
- (b) Is the converse of (a) true? Justify and prove your answer.

Question 3:

Find the following if exist (prove using definition of limit or sequence characterization)

(a)  $\lim_{x \rightarrow 0} \left( \cos \frac{1}{2x} \right)$

(b)  $\lim_{x \rightarrow \infty} \operatorname{sgn}(x)$

(c)  $\lim_{x \rightarrow -\infty} \left( \frac{1}{x} \right)$

Question 4:

Consider the function:

$$f(x) = \begin{cases} \frac{1}{x-3} & x > 3 \\ \frac{1}{x-4} & x \leq 3 \end{cases}$$

- (a) Study the continuity of  $f$  on  $\mathbb{R}$  and specify the discontinuities of  $f$ .
- (b) Are the discontinuities of  $f$  removable? Explain your answer.

BONUS: Find using definition of limit

$$\lim_{x \rightarrow \infty} (-2x).$$