College of Science.
Department of Mathematics

كلية العلوم
قسم الرياضيات

Final Exam
Academic Year 1445 Hijri- First Semester

Exam Information معلومات الامتحان			
Course name	**280**		اسم المقرر
Course Code	**Math**		رمز المقرر
Exam Date	**2023-12-26**	**1445-06-15**	تاريخ الامتحان
Exam Time	**08: 00 AM**		وقت الامتحان
Exam Duration	**3 hours**	ثلاث ساعات	مدة الامتحان
Classroom No.			رقم قاعة الاختبار
Instructor Name	**Dr. Haifa Bin Jebreen**		اسم استاذ المقرر

Student Information معلومات الطالب		
Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Serial Number		الرقم التسلسلي

General Instructions:

- Your Exam consists of PAGES (except this paper)
- Keep your mobile and smart watch out of the classroom.
-

تعليمات عامة:

- عدد صفحات الامتحان 2 صفحة. (بإستثناء هذه الورقة)
- يجب إبقاء الهواتف والساعات الذكية خارج قاعة الامتحان.
-

هذا الجزء خاص بأستاذ المادة
This section is ONLY for instructor

#	Course Learning Outcomes (CLOs)	Related Question (s)	Points	Final Score
1	1.1 Explain fundamental concepts of real Analysis.	Q1		
2	1.2 Describe some properties of functions.	Q5(2)		
3	2.1 Models problems with functions	Q4,Q5(1)		
4	2.2 Solve problems of convergence, limit, continuity and differentiability.	Q2,Q3,Q5(3) Q6		
5				
6				
7				
8				

EXAM COVER PAGE

Question 1

1. Prove that for every real number, there exists an integer n such that $n-1 \leq x < n$. Find such n if $x = -\frac{17}{5}$.

2. Determine $\sup(A)$ and $\inf(A)$ where $A = \{x \in \mathbb{R} : x^2 - 9 < 0\}$, and justify your answer.

Question 2 [4+4]

Use the definition of the limit to find the following if they exists.
1. $\lim_{n\to\infty} \frac{n^3}{2n^4+1}$.
2. $\lim_{n\to\infty} c^{\frac{1}{n}}$, where $c > 1$.
3. $\lim_{n\to\infty} na^n = 0$, where $0 < a < 1$.

Question 3

Discuss the convergence of the following series:
(i) $\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n^2+1}$
(ii) $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

Question 4

1. Find the following limits, if they exist, and prove using the definition of the limit or sequence characterization:
$a)$ $\lim_{x\to 0} \frac{x^2}{|x|}$ (b) $\lim_{x\to\infty} \frac{x^2}{e^x}$.

2. Let

$$f\left(x\right)=\left\{\begin{array}{c} x^2 \text{ if } x\in\mathbb{Q} \\ 0 \text{ if } x\in\mathbb{Q}\end{array}\right.$$

Prove that f is differentiable at $x=0$, and evaluate $f'(0)$.

Question 5

1. Determine a real interval of length $\frac{1}{2}$ where the equation

$$x^3 - 6x^2 + \frac{5}{2} = 0,$$

has a solution. Justify your answer.

2. Prove that if f is continuous on $[a, b]$ and has zero derivative on (a, b), then f is constant.

3.Use Taylor's theorem with $n = 3$ and $x_0 = 0$ to obtain a suitable approximation of the function $f(x) = \sqrt{1-x}$ by a polynomial of degree 3.

Question 6

Let

$$f(x) = \begin{cases} 1 \text{ if } x \in \mathbb{Q} \cap [-2, 2] \\ -1 \text{ if } x \in \mathbb{Q}^c \cap [-2, 2] \end{cases}$$

i) Find the upper and the lower integral of f over $[-2, 2]$.

ii) Is f integrable on $[-2, 2]$? justify your answer.

iii) Is $|f|$ integrable on $[-2, 2]$? justify your answer.