Question I

(a) Prove the Archimedean property; for every $x > 0$, there is a natural number n such that $x > \frac{1}{n}$.

(b) Determine *Sup(A) and Inf(A)* where $A = \left\{1 - \frac{1}{n} : n \in N\right\}$, justify your answer.

(c) Prove that for any real number *x*, there is a sequence of rational numbers converging to *x*.

Question II

(a) Use the definition of the limit to find the following if exists.

(i) $\lim_{n\to\infty} \frac{5n^2+1}{3n^2+3}$.

(ii) $\lim_{n\to\infty} a^n, where\ 0 < a < 1$

(b) If $\lim_{n\to\infty} \frac{x_n-1}{x_n+1} = 0$, then prove that $lim\, x_n$=1.

Question III

Let $x_1 = 1, x_{n+1} = \sqrt{x_n + 3}.\ for\ all\ n \in N.$

(a) Prove (x_n) is monotone.
(b) Prove (x_n) is bounded.
(c) Find the limit of (x_n).

Question IV:

Prove or disprove the following, where (x_n) is a sequence of real numbers.

(a) $Sup(A) = 3, where\ A = (1,3)$.
(b) $\widehat{N}=\emptyset$
(c) Every sequence has a convergent subsequence.
(d) There is an unbounded sequence that has a convergent subsequence.