MAKE UP FINAL EXAMINATION, 2022

Dept. Math., College of Science, KSU

Math: 107 Full Mark: 40 Time: 3 Hours

Q1. [3+2=5] (a) Find all values of x, y, and z for which the matrix A is *symmetric*

$$A = \begin{bmatrix} 2 & x-2y+2z & 2x+y+z \\ 3 & 5 & x+z \\ 0 & -2 & 7 \end{bmatrix}$$

(b) Let

$$B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

and $f(x) = x^2 + 3x + 5$. Find $f(B)$.

Q2. [2+2+5=9] (a) Evaluate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \beta & 1 & 1 \\ 1 & 1 & \delta & 1 \\ 1 & 1 & 1 & \alpha \end{bmatrix}$$

(b) Suppose that D and E are 3×3 matrices with $\det(D) = 4$ such that $D(E - 2D) = 0$, find $\det(E)$.

(c) Let

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Find $(C\ adj(C))$, and hence check if $det\,(C\ adj(C)) = [det(C)]^3$ is true.

Q3. [2+3+2=7] (a) Find the volume of the parallelepiped (box) having $\mathbf{a} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{j} - 2\mathbf{k}$, and $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ as adjacent edges.

(b) Find the distance between the point $A(3, -1, 4)$ and the line given by $x = -2 + 3t$ $y = -2t$, and $z = 1 + 4t$.

(c) Find a set of parametric equations for the line of intersection of the planes: $x - 2y + z = 0$, $2x + 3y - 2z = 0$.

Q4. [2+2+4=8] (a) Find the velocity, speed and acceleration of a particle that moves along the plane curve described by $\mathbf{r}(t) = 2\sin\frac{t}{2}\mathbf{i} + 2\cos\frac{t}{2}\mathbf{j}$.

(b) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = \cos 2t\mathbf{i} - 2\sin t\mathbf{j} + \frac{1}{1+t^2}\mathbf{k}$ that satisfies the initial condition $\mathbf{r}(0) = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

(c) Find the tangential and normal components of acceleration, and curvature for the position vector given by $\mathbf{r}(t) = 3t\mathbf{i} - t\mathbf{j} + t^2\mathbf{k}$.

Q5. [2+3+3+3=11]

(a) Prove that $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + (y-3x)^2}$ does not exist.

(b) Find an equation of the tangent plane to the surface of the equation $x\ln y + y\ln z + xz = 1$ at the point $P_0(1, 1, 1)$.

(c) Find the local extrema and saddle points if any of the function $f(x, y) = x^2 + 4y^2 - x + 2y$

(d) Using Lagrange multipliers find extrema of $f(x, y, z) = x + y + z$ subject to the constraint $x^2 + y^2 + z^2 = 25$.