$\boxed{Q_1}$ $P(A) = 50\%$ $P(D|A) = 1\%$

(a) $P(B) = 35\%$ $P(D|B) = 2\%$

$P(C) = 15\%$ $P(D|C) = 3\%$

$$P(D) = \underbrace{P(A)P(D|A)} + P(B)P(D|B) + P(C)P(D|C)$$

$$= (0.50)(0.01) + (0.35)(0.02) + (0.15)(0.03)$$

$$= 0.005 + 0.007 + 0.0045$$

$$= 0.0165$$

(b) $$P(A|D) = \frac{P(A \cap D)}{P(D)}$$

$$= \frac{0.005}{0.0165} = 0.303$$

$\boxed{Q_2}$ (a) $X \sim \text{Poisson}(\lambda)$

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

$$M_X(t) = E[e^{tX}] = \sum_{x=0}^{\infty} e^{tx} \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t\lambda)^x}{x!}$$

by $e^y = \sum_{x=0}^{\infty} \frac{y^x}{x!}$

$$= e^{-\lambda} e^{e^t\lambda} = e^{\lambda(e^t - 1)}$$

$$M_X'(t) = \frac{d}{dt}\left[e^{\lambda(e^t-1)}\right] = \lambda e^t e^{\lambda(e^t-1)}$$

$$\Rightarrow E[X] = M_X'(0) = \lambda e^0 e^{\lambda(e^0-1)} = \lambda$$

(b)

$\lambda = 3$

X = no. of claims $\sim$ Poisson(3)

$P(X \geq 4) = ?$

$$P(X \geq 4) = 1 - P(X < 4)$$
$$= 1 - P(X \leq 3)$$
$$= 1 - \{P(X=0) + P(X=1) + P(X=2) + P(X=3)\}$$

use $P(X=k) = \frac{e^{-\lambda}\lambda^k}{k!}$, $\lambda = 3$

$$= 1 - 13e^{-3}$$

$\boxed{Q_3}$ $X \sim$ log-normal dist. , $P_r(X>35)$

$$E[X] = 30 \quad Var[X]=5$$

$$P_r(X>35) = 1 - P_r(X \leq 35)$$
$$= 1 - P_r(\ln X \leq \ln 35)$$

Find μ and σ :

$$E[X] = e^{\mu+\sigma^2/2} = 30 \longrightarrow ①$$

$$Var[X] = e^{2\mu+\sigma^2}(e^{\sigma^2}-1) = 5 \longrightarrow ②$$

from ① $\Rightarrow \mu + \sigma^2/2 = \ln 30$ ③

put ③ in ② :

$$2\mu + \sigma^2 = 2\ln 30$$

$$\Rightarrow e^{2\ln 30}(e^{\sigma^2}-1) = 5$$

$$\Rightarrow e^{\ln 900}(e^{\sigma^2}-1) = 5$$

$$\Rightarrow (e^{\sigma^2}-1) = \frac{5}{900}$$

$$\Rightarrow e^{\sigma^2} = 1 + \frac{5}{900}$$

$$\Rightarrow \sigma^2 = \ln\left(1+\frac{5}{900}\right)$$

$$\approx 0.0055$$

put in ③ to get

$$\mu = \ln 30 + \ln\left(1+\frac{5}{900}\right)/2 \approx 3.4040$$

$\boxed{Q_3}$ Continue

$$Pr(X > 35) = 1 - \Phi\left[\frac{\ln 35 - 3.4040}{\sqrt{0.0055}}\right]$$

$$= 1 - \Phi(2.04) \quad \text{find from table}$$

$$\Rightarrow Pr(X > 35) = 1 - 0.9793 \approx 0.02$$

$\boxed{Q_4}$

(a) $$\int_{1/2}^{1}\int_{0}^{y} (6 - 6y)\, dx\, dy$$

$$= \int_{1/2}^{1} (6 - 6y)\, y \; dy$$

$$= \int_{1/2}^{1} 6y - 6y^2 \; dy$$

$$= 6\frac{y^2}{2} - 6\frac{y^3}{3}\Big|_{1/2}^{1} = \frac{1}{2}$$

(b) $$f_X(x) = \int_x^1 6(1-y)\,dy = 6y - 3y^2\Big|_x^1 = \begin{cases} 3x^2 - 6x + 3, & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \int_0^y 6(1-y)\,dy = 6\left[y - \frac{y^2}{2}\right]_0^y = \begin{cases} 6y(1-y), & 0 < y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Q4 Continue

(c) $f_X(x)\, f_Y(y) \neq f_{X,Y}(x,y)$

So X, Y are dependendet.

Q5

X = loss amounts ~ uniform[0,450]

$$Y = \begin{cases} 0 & 0 \leq X < d \\ X-d & d \leq X < 450 \end{cases}$$

density fn

$f_X(x) = \frac{1}{450}$

$$E[Y] = 120 = \int_d^{450} (x-d) \cdot \frac{1}{450}\, dx$$

$$\Rightarrow 120 = \left. \frac{(x-d)^2}{2*450} \right|_d^{450}$$

Solve for d $\Rightarrow$ $\boxed{d = 121.37}$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = 1$$

Q6 (a) $\int_0^1 \int_0^y c\,x\,y \; dx\, dy = 1$

$\Rightarrow \; = \int_0^1 \left(c y \frac{x^2}{2} \right)\Big|_0^y dy = \int_0^1 c \frac{y^3}{2} dy$

$= \frac{c}{2} \frac{y^4}{4}\Big|_0^1 = 1 \Rightarrow \boxed{c = 8}$

(b) $f_X(x) = \int_0^x 8xy\,dy = 8x \frac{y^2}{2}\Big|_0^x = 4x^3$

$f_{Y|X}(y|x) = \frac{8xy}{4x^3} = \frac{2y}{x^2}$

(c) $E[Y \mid X = x]$

$= \int_0^x y \cdot f_{Y|X}(y|x)\, dy$

$= \int_0^x y \cdot \frac{2y}{x^2}\, dy = \frac{2}{3} x \;, \; 0 \leq x \leq 1$