King Saud University, Department of Mathematics
Math 204 (2H), 30/100, Mid term Exam S2. 41/42

Question 1[4,3] a) Find and sketch the largest region in $\mathbb{R}^2$, for wich the following initial value problem admits a unique solution

$$\begin{cases} \left(2\sqrt{y} + \sqrt{x+y}\right) dx - \ln(1-x^2)dy = 0 \\ y(-\frac{1}{2}) = 1. \end{cases}$$

b) Solve the differential equation

$$\tan y - x\frac{dy}{dx} = 4x^2 \tan y, \quad y \in (0, \pi), \ x > 0.$$

Question 2 [3,3] a) Find the general solution of the differential equation

$$\left(x \cos\frac{y}{x} + y\right) dx - ydy = 0.$$

b) Use the substitution $u = \ln y$ to reduce the differential equation

$$x\frac{dy}{dx} = 2x^2 y + y \ln y, \quad x > 0, \ y > 0.$$

to a linear equation, and then solve it.

Question 3. [3,3]. a) Solve the initial value problem

$$\begin{cases} \frac{dy}{dx} = 3 - \sqrt{x+y-1} \\ y(0) = 1. \end{cases}$$

Question 4 [3,3]. b) Obtain the general solution of the following differential equation

$$(ye^{-2x} + y^3)dx - e^{-2x}dy = 0.$$

Question 5 [5] Initially there were 60 grams of a radioactive material present. After 8 hours the mass decreases by 4%. We suppose that the rate of decay is proportional to the amount of the material at time t. Determine the half life of this material.