Midterm Exam
Math 280
3^{rd} semester 1444

The first question.[3+3]

1. Let A be a nonempty subset of $\mathbb{R}$. If A is bounded below, show that $-A$ is bounded above and $\inf A = -\left(\sup -A\right)$.

2. If x and y are two real numbers and $x < y$, prove that there exists a rational number r such that
$$x < r < y.$$

The second question[3+3]

1. Prove using the definition that $\lim_{n\to\infty} \frac{2n+3}{5n+1} = \frac{2}{5}$.

2. Prove that if $x_n \to x$ and $y_n \to y$, then $x_n + y_n \to x + y$.

3. If $\lim_{n\to\infty} \frac{x_n-1}{x_n+1} = 0$, prove that $\lim_{n\to\infty} x_n = 1$.

The third question[3+3]

1) Let $f : (-1, 1) \to \mathbb{R}$ satisfying
$$|f(x) - 2| \leq 2|x - 1| \quad \text{for all} \quad x \in \mathbb{R}.$$
Prove that f is continuous at $x = 1$.

2) Show that: $f(x) = (1 + x)^2$ is not uniformly continuous on $\mathbb{R}$.

The forth question.[2+2+2]
Test the following series for convergence:

1. $\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)}$,

2. $\sum_{n=0}^{\infty} \frac{n}{2^n}$,

3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.