

106Midterm (Sem2/42)

Question1

a) $P=\{0,1,2,3,4\}$

$$S_4 = \frac{4-0}{3.4} \{\sqrt{8} + 12 + 8 + 4\sqrt{35} + \sqrt{72}\} \quad (1,5)$$
$$= 18.326 \quad (0.5)$$

b) $\int \frac{e^x \ln(e^x+1) dx}{e^x+1} = \int u du \quad u = \ln(1+e^x) \quad (2)$

$$= \frac{1}{2} (\ln(1+e^x))^2 + C \quad (1)$$

c) $y = (1+x^2)^{2x+1}$

$$\frac{y'}{y} = 2 \ln(1+x^2) + \frac{2x(2x+1)}{1+x^2} \quad (1,5)$$

$$y' = \left(2 \ln(1+x^2) + \frac{2x(2x+1)}{1+x^2}\right) y \quad (0,5)$$

Question2

a) $\int \frac{2^x dx}{\sqrt{4-4^x}} = \frac{1}{\ln 2} \int \frac{du}{\sqrt{2^2-u^2}} \quad u = 2^x \quad (2)$

$$= \frac{1}{\ln 2} \sin^{-1} \left(\frac{2^x}{2} \right) + C \quad (1)$$

b) $\int \frac{x^4 dx}{\sqrt{x^{10}-1}} = \frac{1}{5} \int \frac{du}{\sqrt{u^2-1}} \quad u = x^5 \quad (1)$

$$= \frac{1}{5} \cosh^{-1}(x^5) + C \quad (1)$$

c) $\int \frac{dx}{x\sqrt{1-x^6}} = \frac{1}{3} \int \frac{du}{u\sqrt{1-u^2}} \quad u = x^3 \quad (2)$

$$= \frac{-1}{3} \operatorname{sech}^{-1}(x^3) + C \quad (1)$$

Question3

$$a) \lim_{x \rightarrow \infty} \frac{\ln(1+3x)}{x} = \lim_{x \rightarrow \infty} \frac{3}{1+3x} = 0 \quad (2)$$

$$\text{so } \lim_{x \rightarrow \infty} (1+3x)^{\frac{1}{x}} = 1 \quad (1)$$

$$b) \int e^x \sin 4x dx = e^x \sin 4x - 4 \int e^x \cos 4x dx \quad (1)$$

$$\int e^x \cos 4x dx = e^x \cos 4x + 4 \int e^x \sin 4x dx \quad (1)$$

$$\text{so } \int e^x \sin 4x dx = \frac{1}{17} e^x (\sin 4x - 4 \cos 4x) + C \quad (1)$$

$$c) \int (\cos x)^5 (\sin x)^4 dx = \int (1-u^2)^2 u^4 du \quad u = \sin x \quad (2)$$

$$= \frac{(\sin x)^9}{9} - \frac{2}{7} (\sin x)^7 + \frac{1}{5} (\sin x)^5 + C \quad (1)$$

Question4

$$a) \int x^3 \sqrt{x^2 - 4} dx = 32 \int (\sec \theta)^4 (\tan \theta)^2 d\theta \quad x = 2 \sec \theta$$

$$= \int (1+u^2)u^2 du \quad u = \tan \theta \quad (2)$$

$$= \frac{32}{5} (\tan \theta)^5 + \frac{32}{3} (\tan \theta)^3 + C$$

$$= \frac{1}{5} (\sqrt{x^2 - 4})^5 + \frac{4}{3} (\sqrt{x^2 - 4})^3 + C \quad (1)$$

$$b) \frac{2x^2 - 11x + 9}{x^3 - 6x^2 + 9x} = \frac{1}{x} + \frac{1}{x-3} - \frac{2}{(x-3)^2} + C \quad (1, 5)$$

$$\int \frac{2x^2 - 11x + 9}{x^3 - 6x^2 + 9x} dx = \ln|x| + \ln|x-3| + \frac{2}{x-3} + C \quad (1, 5)$$