

King Saud University,
College of Sciences
Mathematical Department.

Mid-Term Exam /S2/2022
Full Mark:30. Time 2H
23/03/2022

Question 1. [5, 4] **a)** A radioactive substance has a half-life of 4000 years. If 200 grams were initially present, how much of the substance will be left after 10000 years.

b) Find the general solution of the differential equation

$$dy + \frac{y(x+y)}{x^2} dx = 0, \quad x > 0.$$

Question 2. [5] Find and sketch the largest region of the xy -plane for which the initial value problem

$$\begin{cases} \cos^{-1}(1+y)dx + (\ln(x+1) - 1) dy = 0 \\ y(0) = -1, \end{cases}$$

has a unique solution.

(Hint: $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$)

Question 3. [4, 4]. **a)** Solve the differential equation

$$y \frac{dy}{dx} e^{y-x} - \ln(1 + e^x) = 0.$$

b) By using an appropriate integrating factor, find the general solution of the differential equation

$$\cos x dx + \left(2 + \frac{3}{y}\right) \sin x dy = 0, \quad 0 < x < \pi, \quad y > 0.$$

Question 4. [4, 4]. **a)** Solve the initial value problem

$$\begin{cases} y(y-1) \sin x dx - dy = 0 \\ y\left(\frac{\pi}{2}\right) = 1 \end{cases}$$

b) Determine whether the following functions

$$f_1(x) = \ln(x+2), \quad f_2(x) = \ln(2-x), \quad f_3(x) = \ln(4-x^2),$$

are linearly dependent or linearly independent on the interval $(-2, 2)$.